A Markov chain Monte Carlo approach to confirmatory item factor analysis

Michael C. Edwards
The Ohio State University

An MCMC approach to CIFA

Overview

- Motivating examples
- Intro to Item Response Theory (IRT)
- Review of IRT estimation history
- Current estimation challenges
- Markov chain Monte Carlo (MCMC)
- MCMC applied to IRT
- Some Results

Motivating Examples

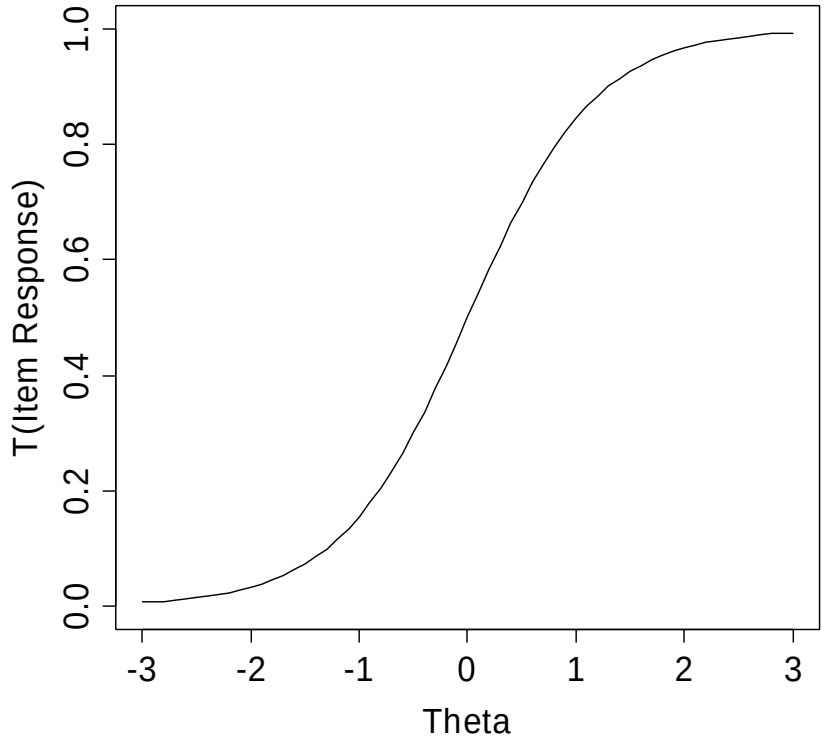
- SAT
  - Diagnostic Scores
- Quality of Life Measurement
  - Dimensionality
  - Scoring

Item Response Theory

IRT is a collection of latent variable models that explain the process by which people respond to items in terms of item and person parameters.

2-Parameter Normal Ogive Model

- One of the most widely used
- Appropriate for dichotomous item responses when guessing is not present

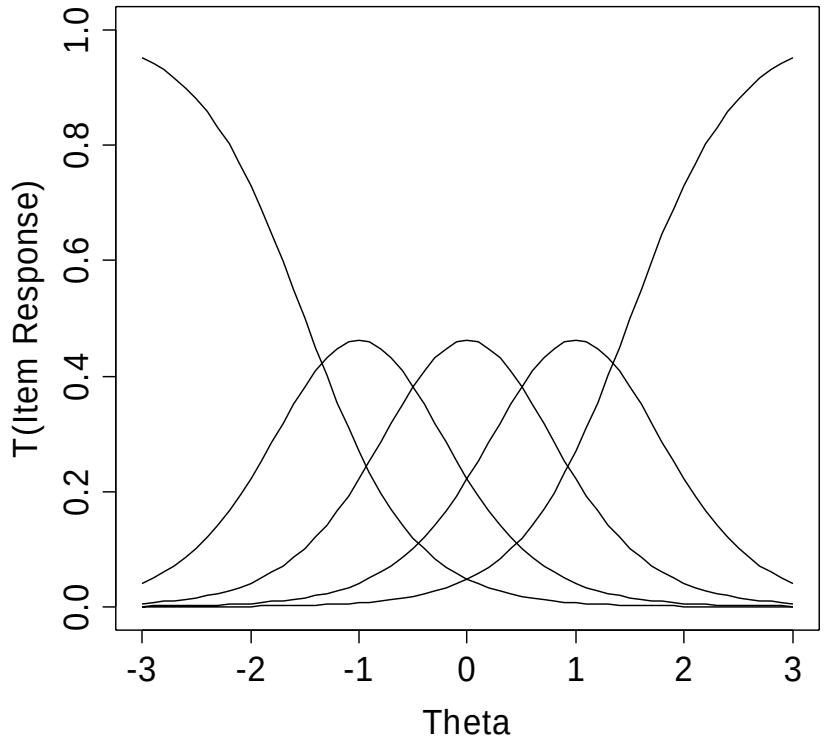


$$P(y_j = 1 | \theta) = \Phi[a_j(\theta - b_j)]$$

$$P(y_j = 1 | \theta) = \Phi[a_j\theta - d_j]$$

Samejima's Graded Model

- Widely used in psychology
- Appropriate for ordered categorical data



$$P(y_j = c \mid \theta) = \Phi[a_j(\theta - b_{jc})] - \Phi[a_j(\theta - b_{jc+1})]$$

$$P(y_j = c \mid \theta) = \Phi[a_j\theta - (d_j + o_{jc})] - \Phi[a_j\theta - (d_j + o_{jc+1})]$$

IRT Parameter Estimation

- Heuristic Estimation
  - Lord (1952), Lord & Novick (1968)
- Joint Maximum Likelihood
  - Lord (1953), Rasch (1960), Birnbaum (1968)
- Maximum Marginal Likelihood
  - Bock & Lieberman (1970)
- Maximum Marginal Likelihood with an EM algorithm
  - Bock & Aitkin (1981)
- Bayes Modal Estimation with an EM algorithm
  - Mislevy (1986)

Potential Uses

- Item Analysis
- Scale Development
- Scoring
  - Linking and Equating
  - Computerized Adaptive Testing (CAT)

Moving to Multiple Dimensions

- TESTFACT
  - Exploratory item factor analysis for dichotomous models
  - Bi-factor model
  - Uses MML-EM and different methods of numerical integration
- SEM approaches
  - WLS, DWLS, UBN, etc.

Old Problems, New Solutions

- “Curse of Dimensionality”

Old Problems, New Solutions

- “Curse of Dimensionality”
- How to handle high dimensional integration?

Markov Chain Monte Carlo

- MCMC estimation can be thought of as Monte Carlo integration using Markov chains
- Monte Carlo integration works by simulating samples from a target distribution and then computing averages to replace expectations
- Simulated values from target distribution - generated by constructing a Markov chain with the target as its stationary distribution

How to Simulate from Target Distribution?

- Metropolis Hastings
- Gibbs Sampling
- Data Augmented Gibbs Sampling (DAG)
- Metropolis Hastings within Gibbs (MHwG)

MCMC and IRT

- Albert (1992)
  - First published application of MCMC to IRT
- Patz & Junker (1996)
  - MHwG for IRT models

- Early forays into MCMC for MIRT
  - Béguin & Glas (2001); Segall (2002)
  - Shi & Lee (1998); Arminger & Muthén (1998)

Dissertation Research

- An MCMC Approach to Confirmatory Item Factor Analysis

Comparing MCMC Approaches

- DA-Gibbs?
- MHwG?
- Combinations?

MCMC with GRM

Authors	*a*	*d*	*o*
A & C (93)	DAG	DAG	DAG
Cowles (96)	DAG	DAG	MHwG
J & A (99)	DAG	DAG	MHwG
Fox (04)	DAG	-	MHwG
W et al (02)	MHwG	MHwG	MHwG

MCMC with 3PNO

Authors	*a*	*d*	*g*
P & J (99)	MHwG	MHwG	MHwG
W et al (00)	MHwG	MHwG	MHwG
B & G (01)	DAG	DAG	DAG
Segall (02)	DAG	DAG	DAG
Sahu (02)	DAG	DAG	DAG

Performance Components

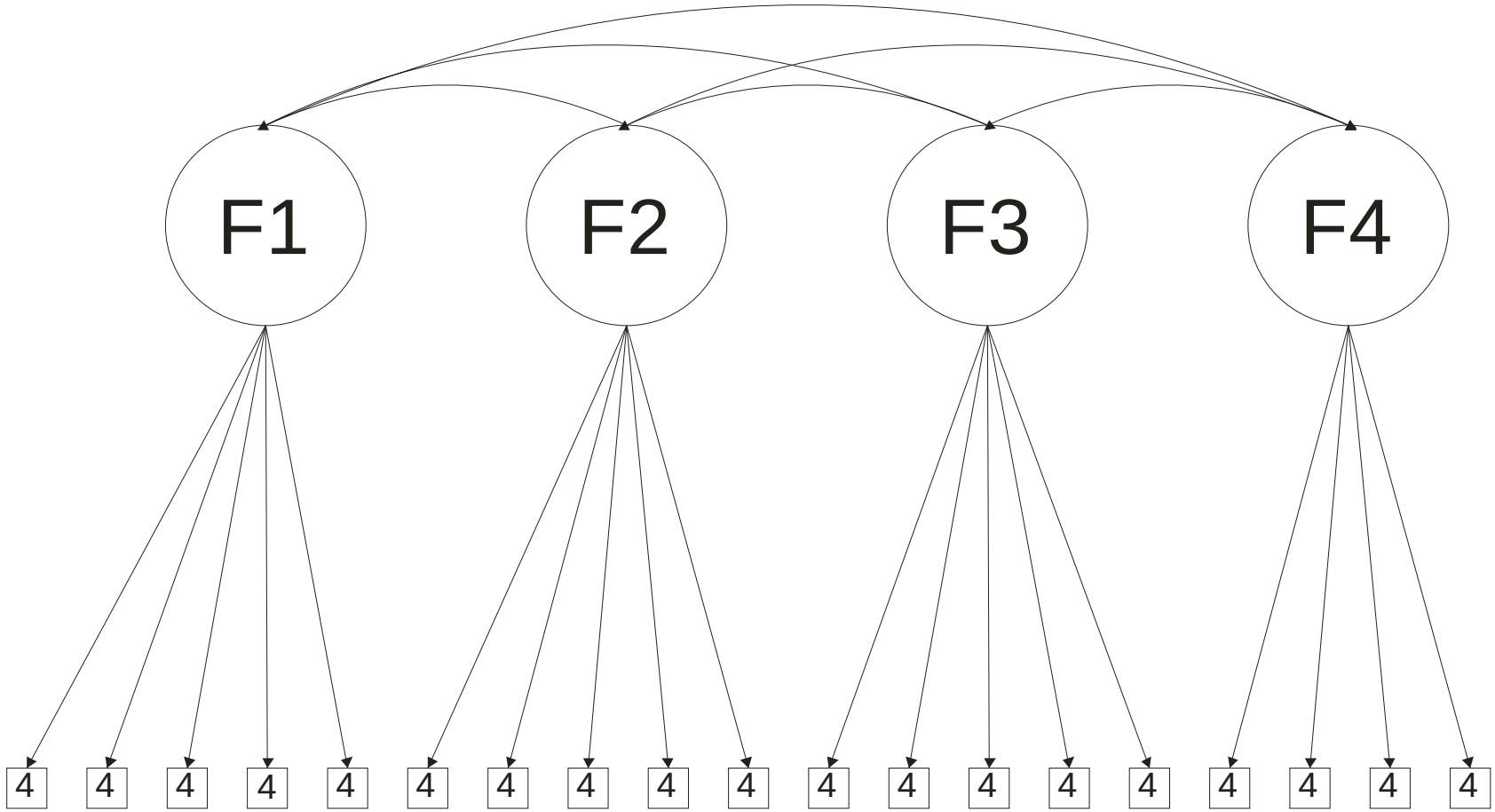
- Examine:
  - Parameter recovery
  - Autocorrelations and effective sample size
  - Mixing of chains
  - Time per cycle

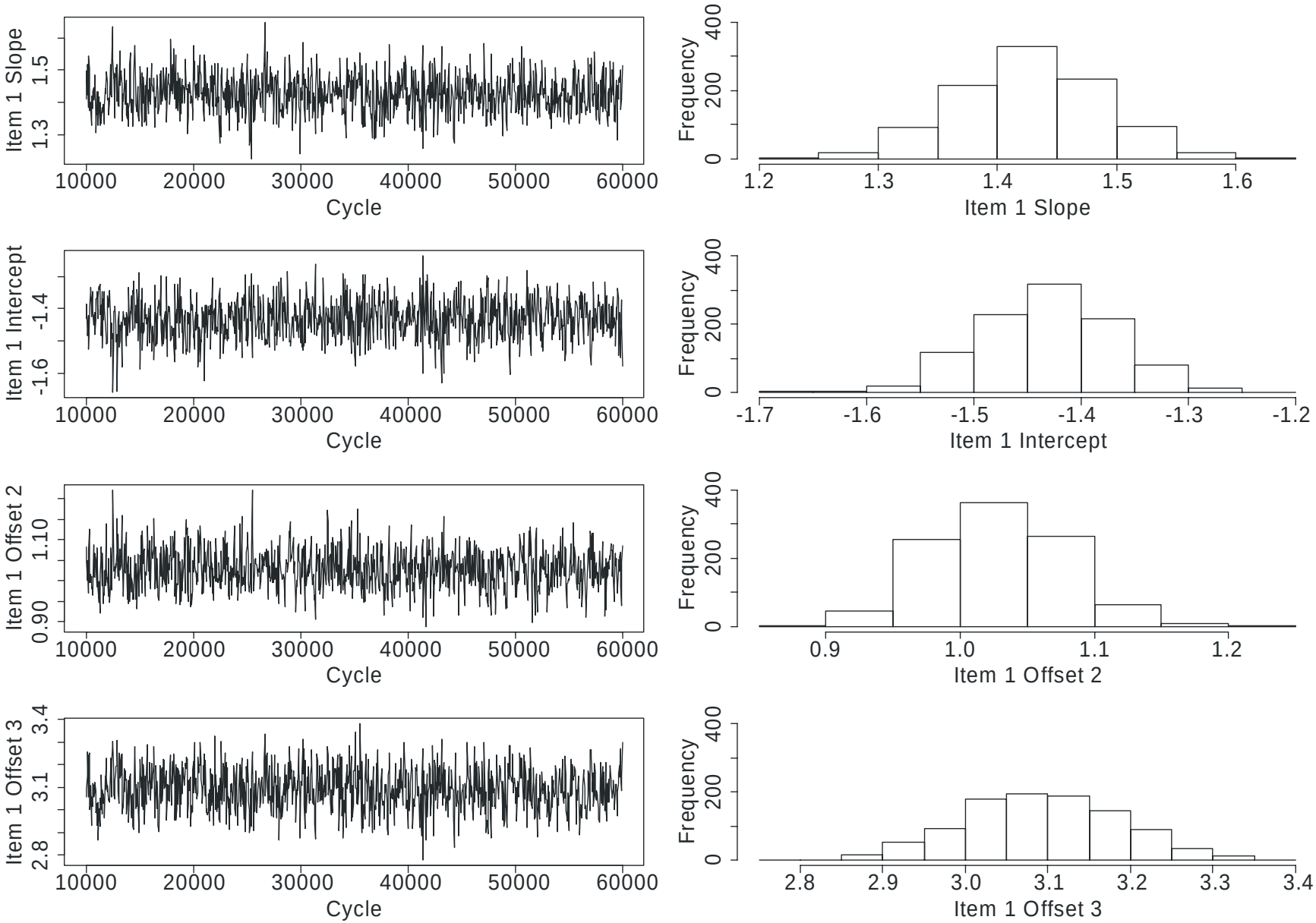
Next Steps

- Make it work, make it fast, make it pretty

- R too slow, move to C++
- Multiple correlated factors with independent clustering
- Cross loadings
- Mixed item types

Example 1




Item 1 Slope
1.3
1.5
10000
20000
30000
40000
50000
60000
Cycle
Frequency
0
200
400
1.2
1.3
1.4
1.5
1.6
Item 1 Slope
Item 1 Intercept
-1.6
-1.4
Cycle
-1.7
-1.6
-1.5
-1.4
-1.3
-1.2
Item 1 Intercept
Item 1 Offset 2
0.90
1.10
Item 1 Offset 3
2.8
3.1
3.4
2.9
3.0
3.2
3.3

Example 1 Results (RMSE)

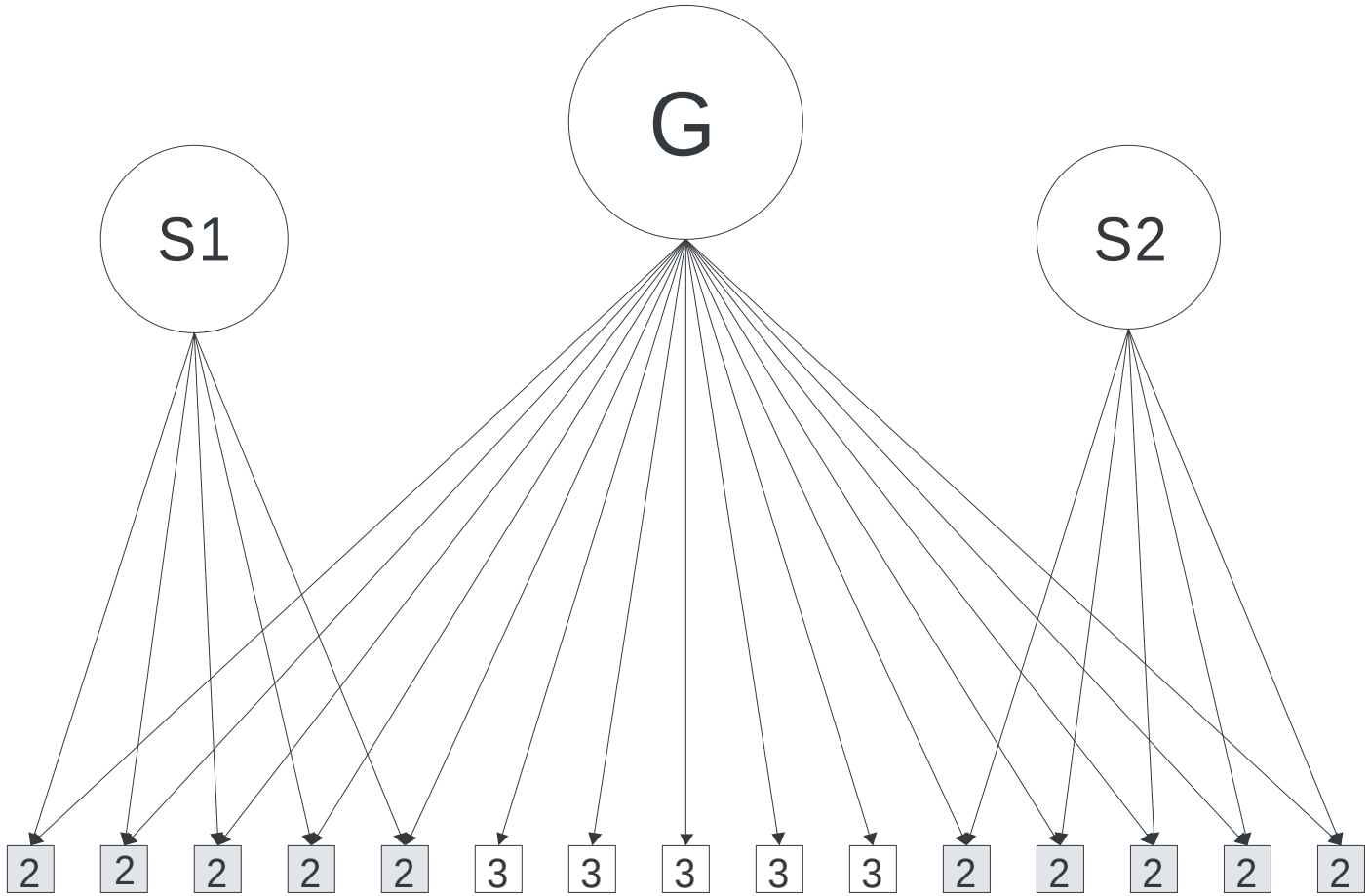
N = 2000

	MCMC	Multilog	WLS
a	0.05	0.05	0.06
d	0.08	0.10	0.10
o_2	0.04	0.05	0.06
o_3	0.07	0.08	0.10
r	0.02	-	0.03

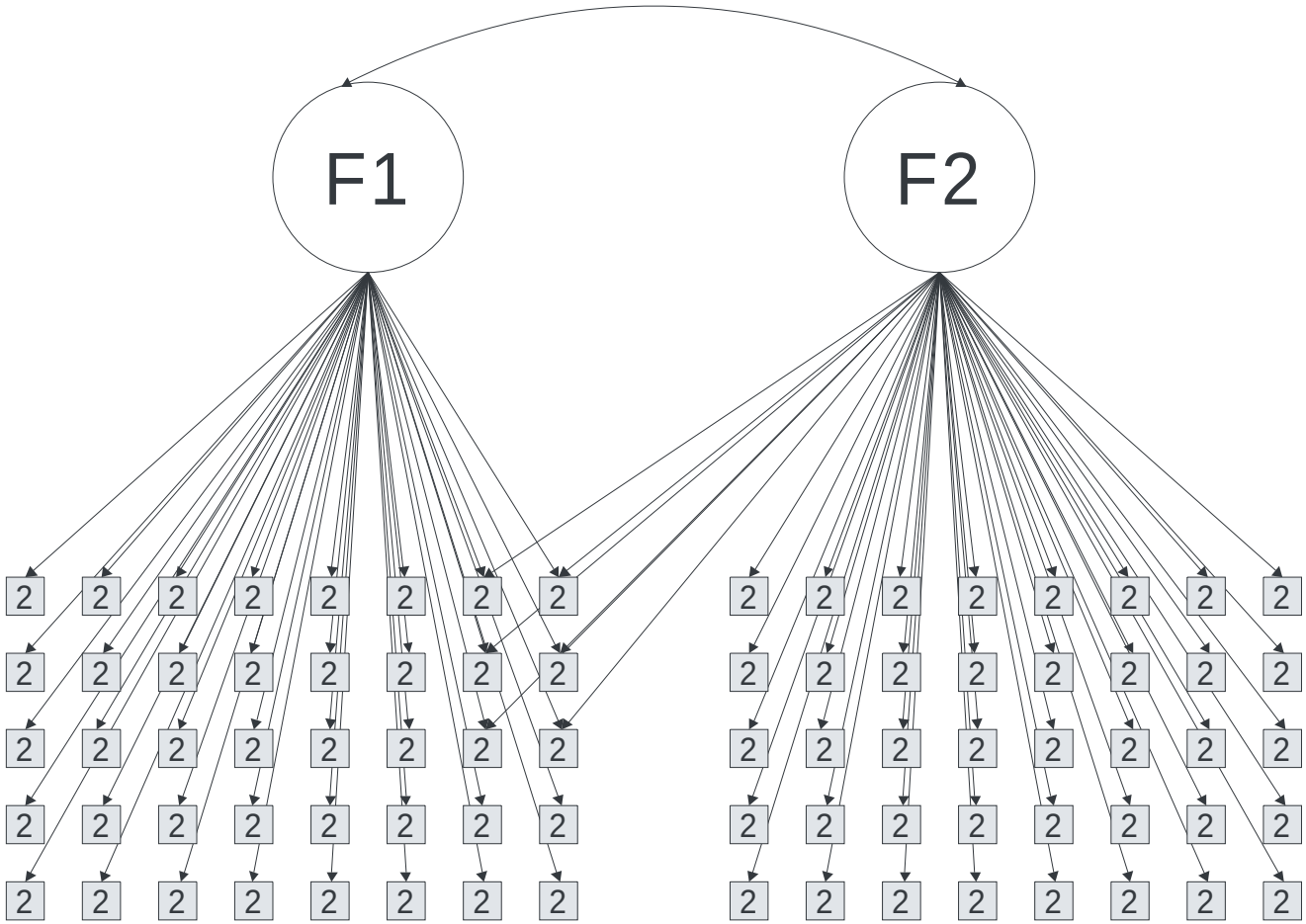
Example 1 Results (RMSE)

	N = 2000			N = 500		
	MCMC	Multilog	WLS	MCMC	Multilog	WLS
a	0.05	0.05	0.06	0.09	0.11	0.27
d	0.08	0.10	0.10	0.10	0.12	0.18
o_2	0.04	0.05	0.06	0.10	0.10	0.18
o_3	0.07	0.08	0.10	0.15	0.16	0.32
r	0.02	-	0.03	0.03	-	0.09

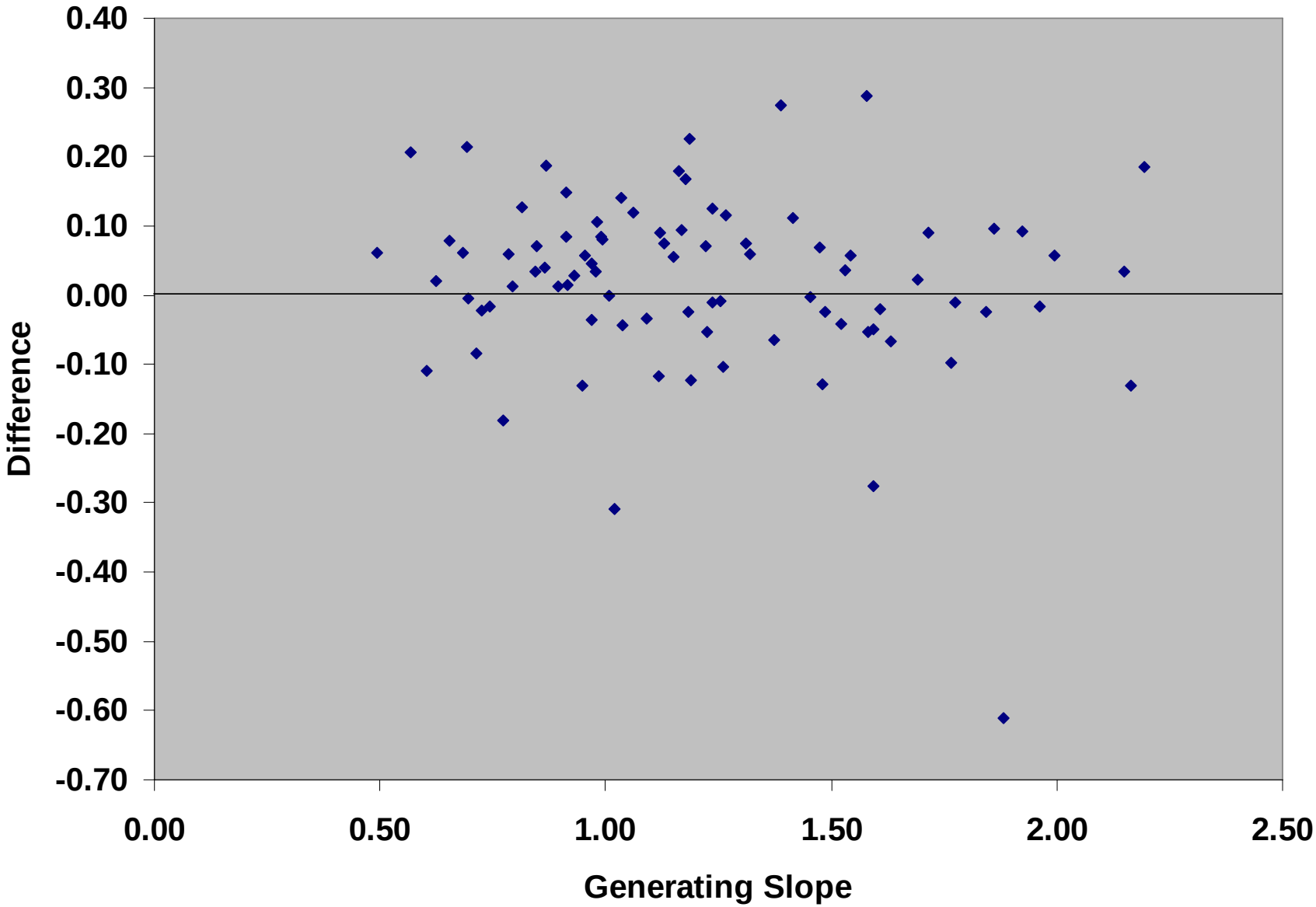
Example 2



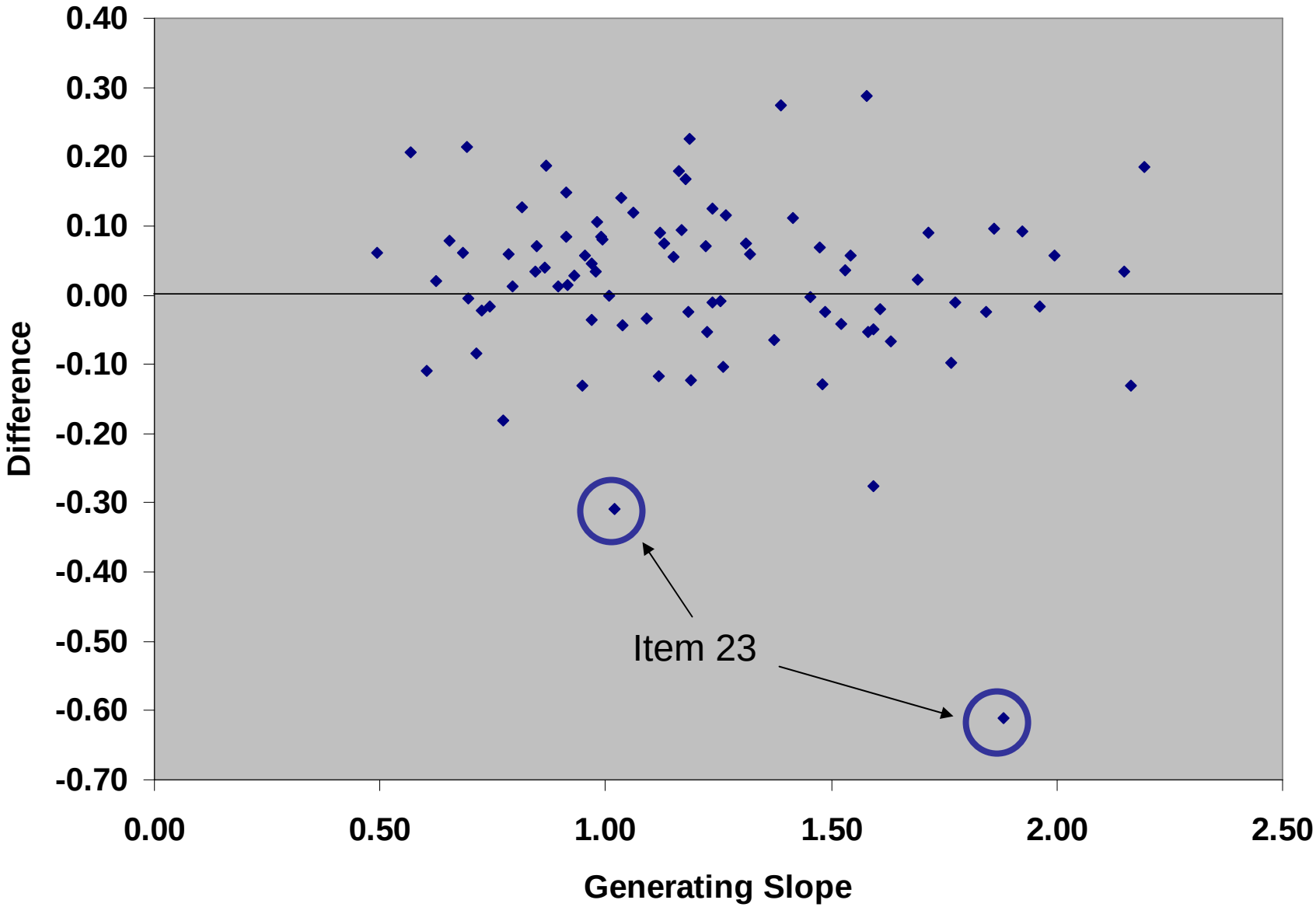
Example 3



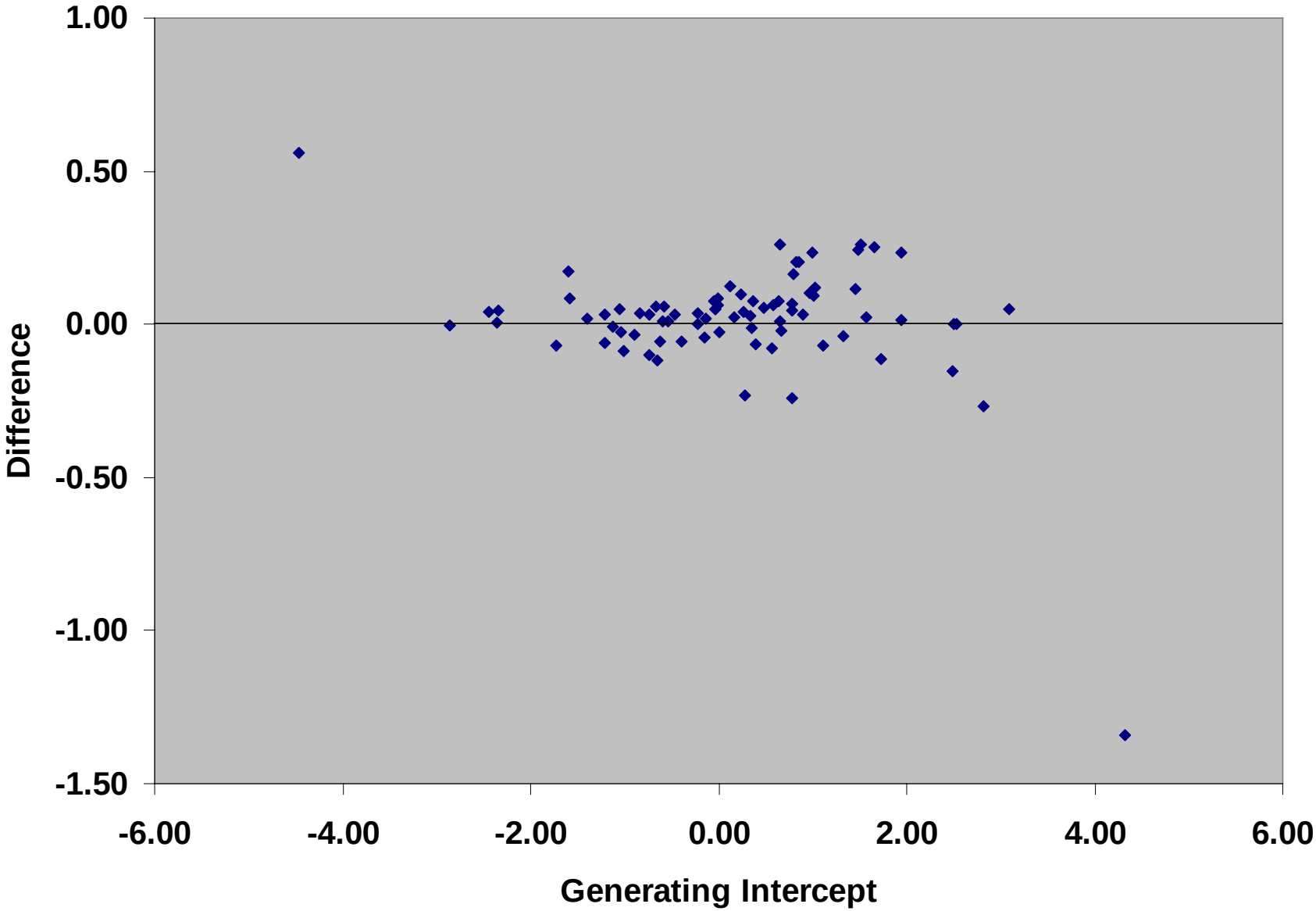
Example 3 - Slopes



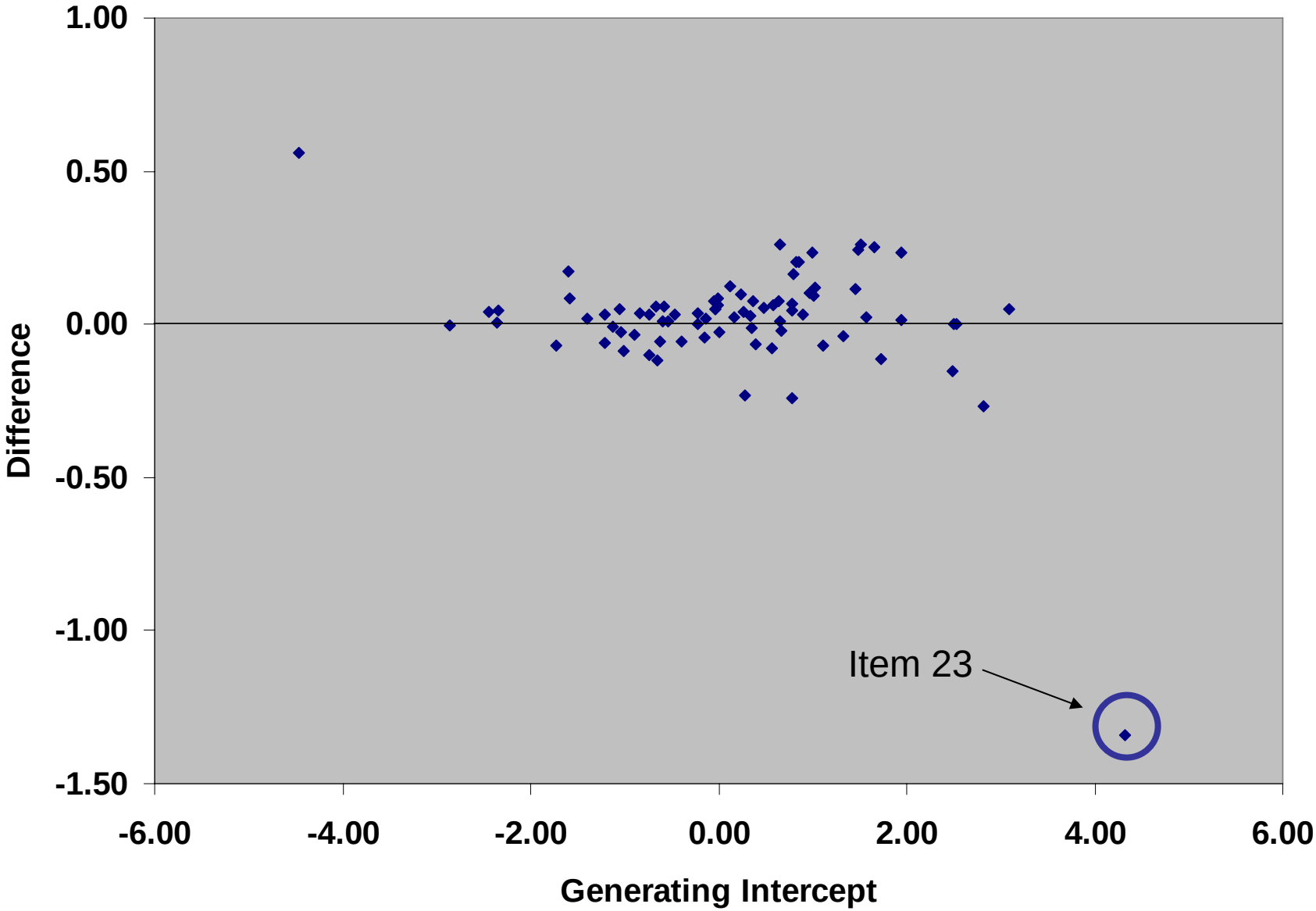
Example 3 - Slopes



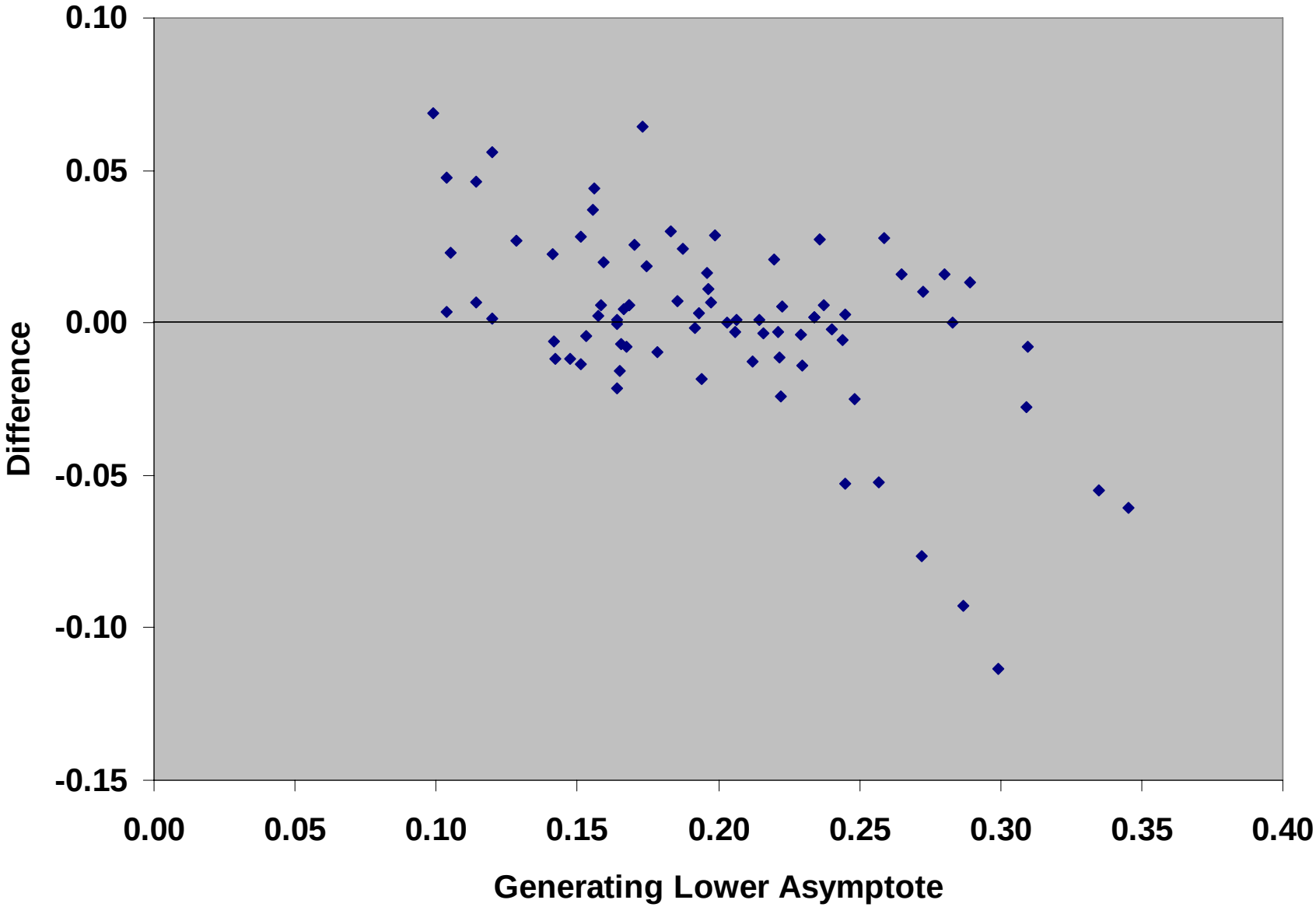
Example 3 – Intercepts



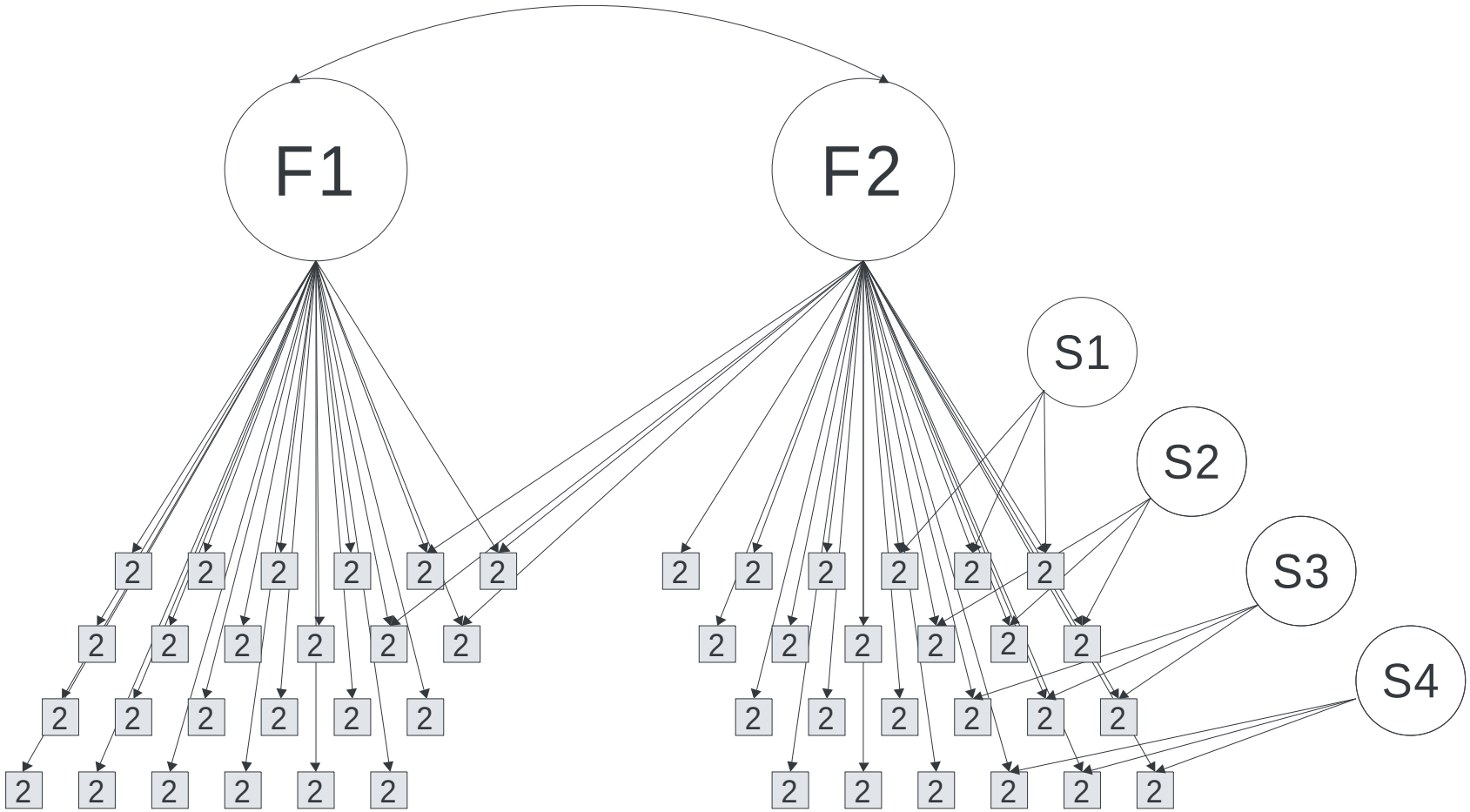
Example 3 – Intercepts



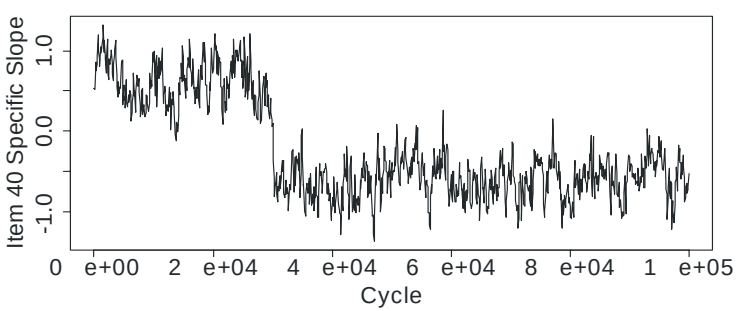
Example 3 – Lower Asymptotes

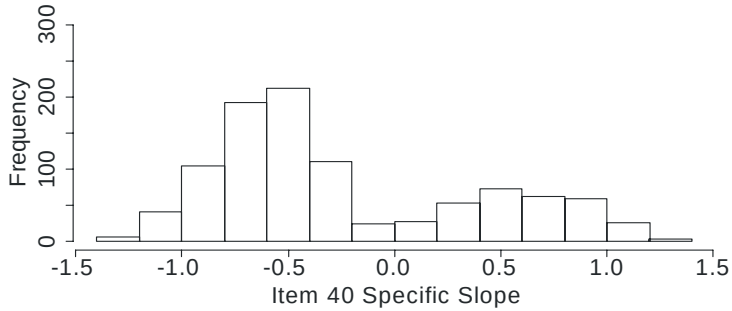


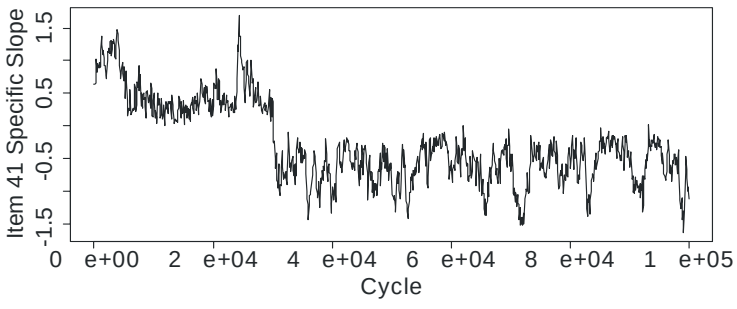
Example 4

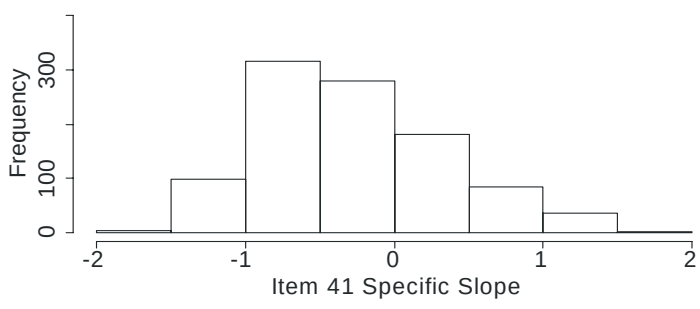


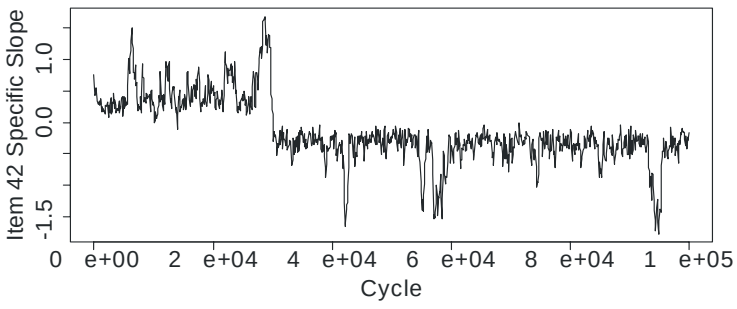
Example 4 - Reflection

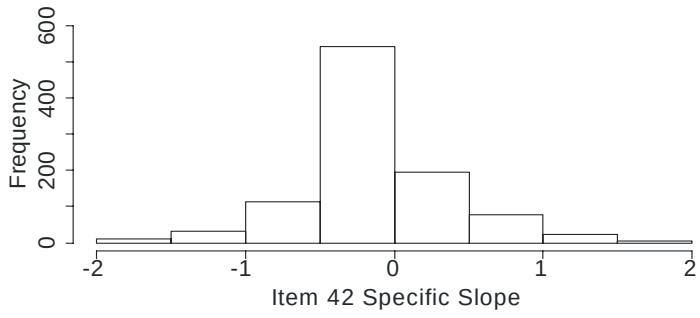




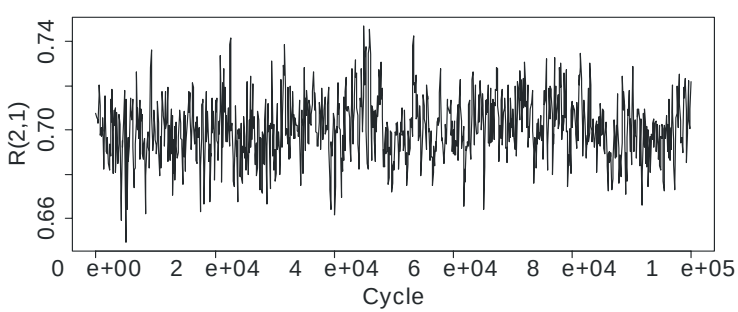


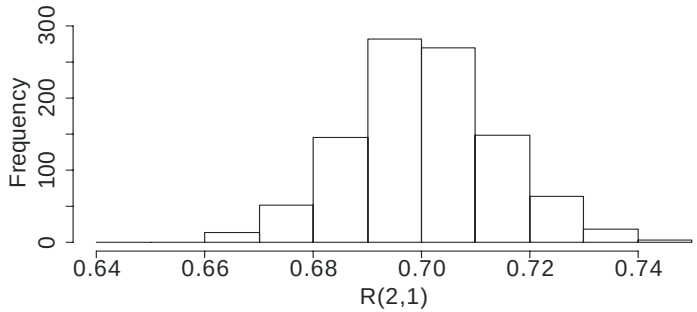


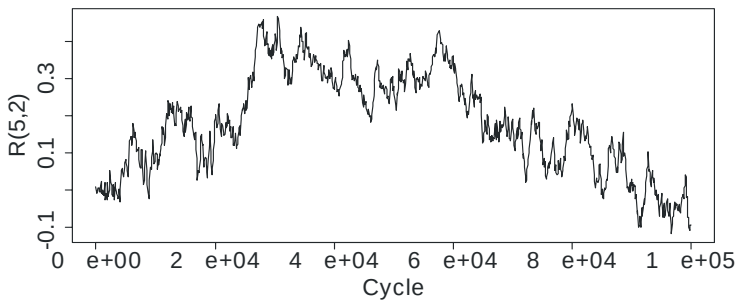


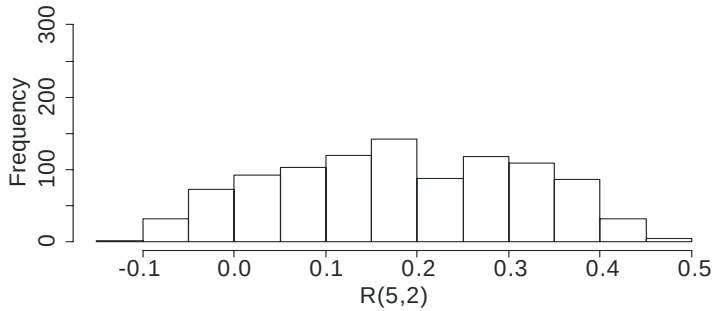


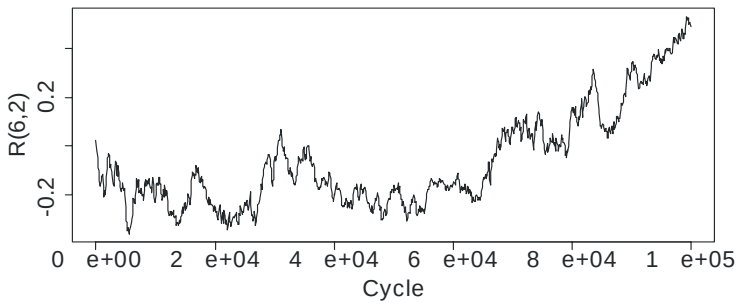
Example 4 – Correlation Troubles

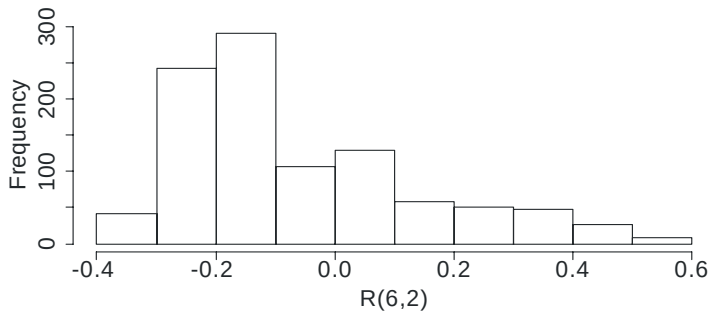




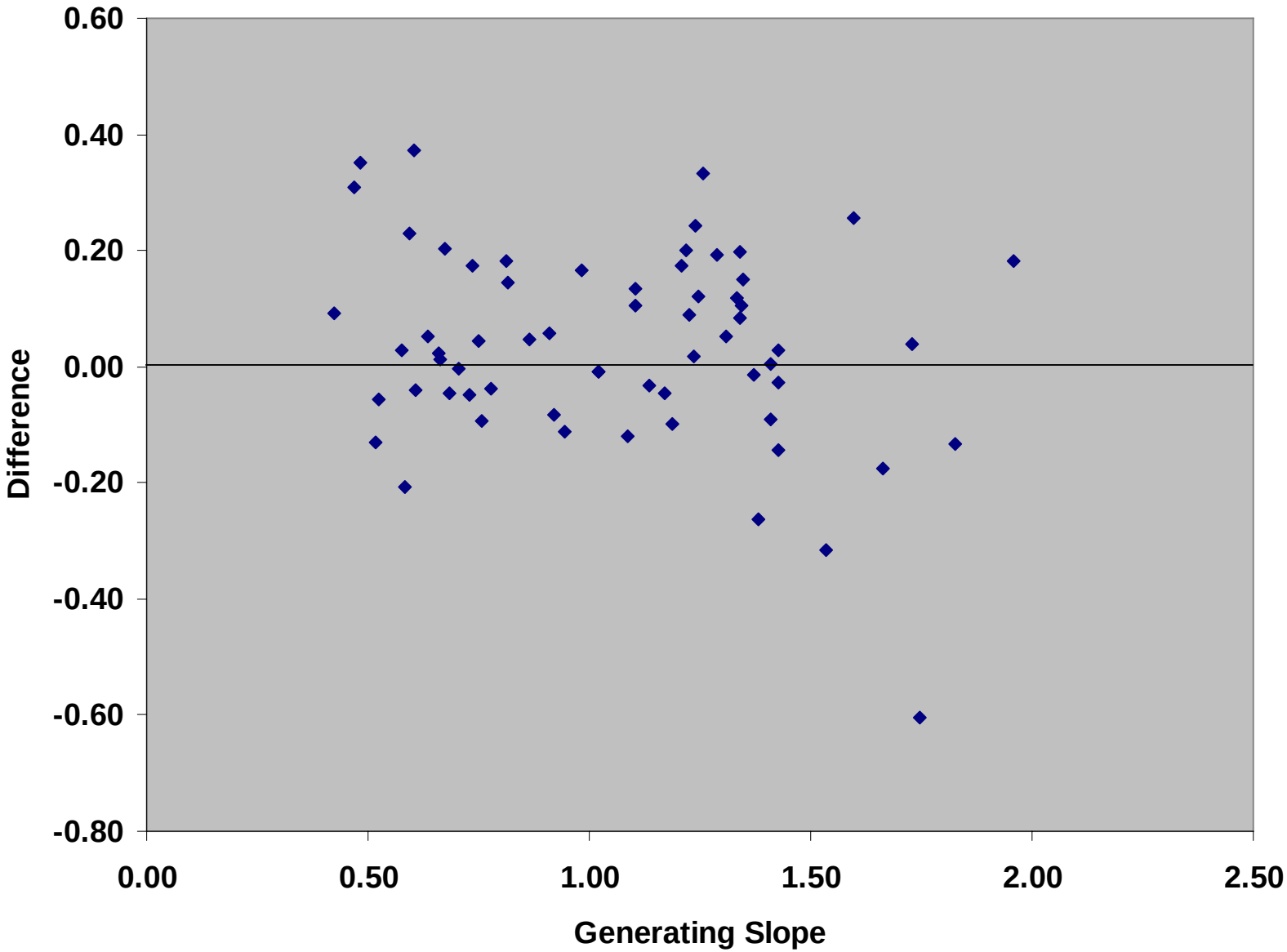




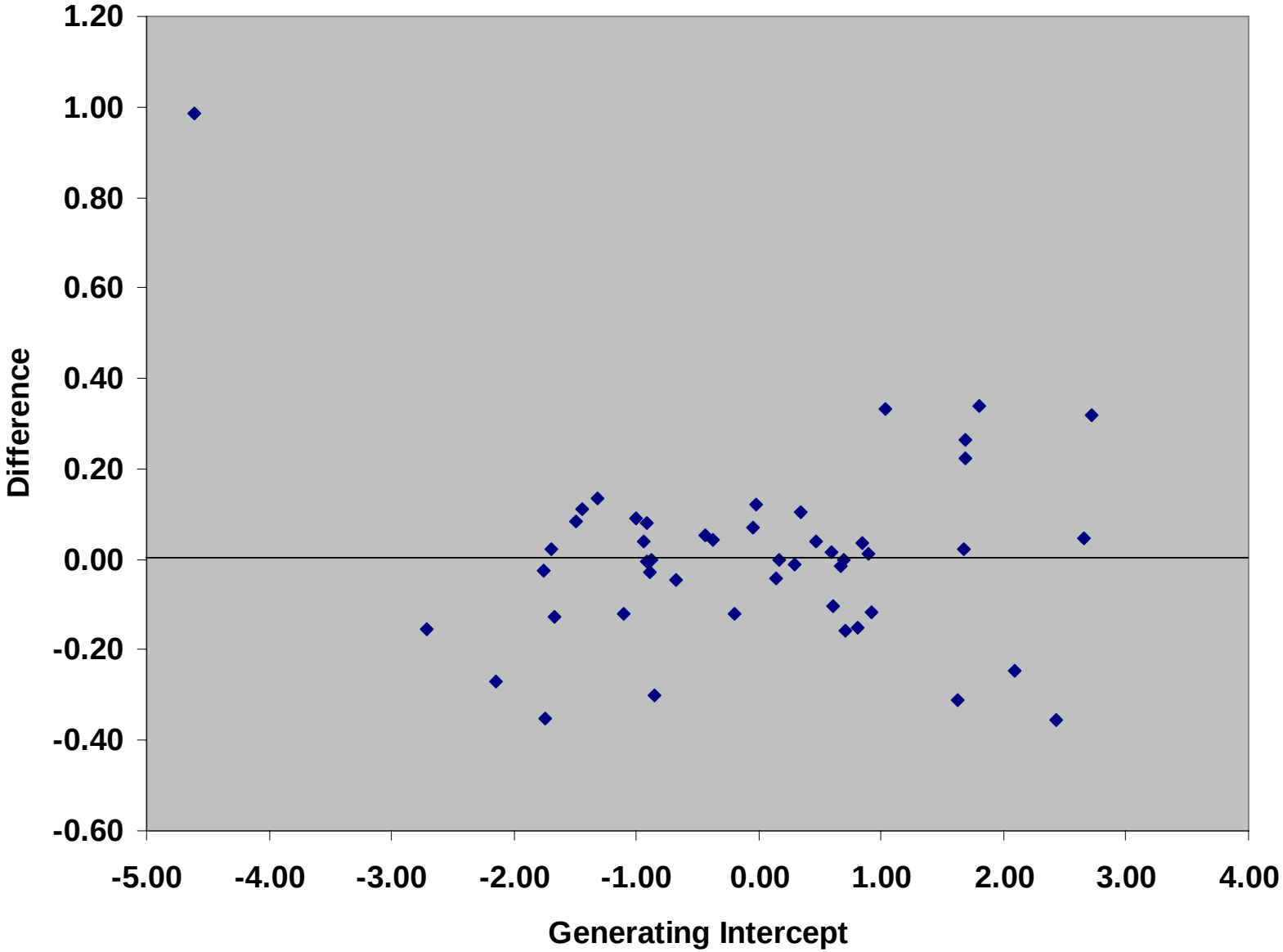




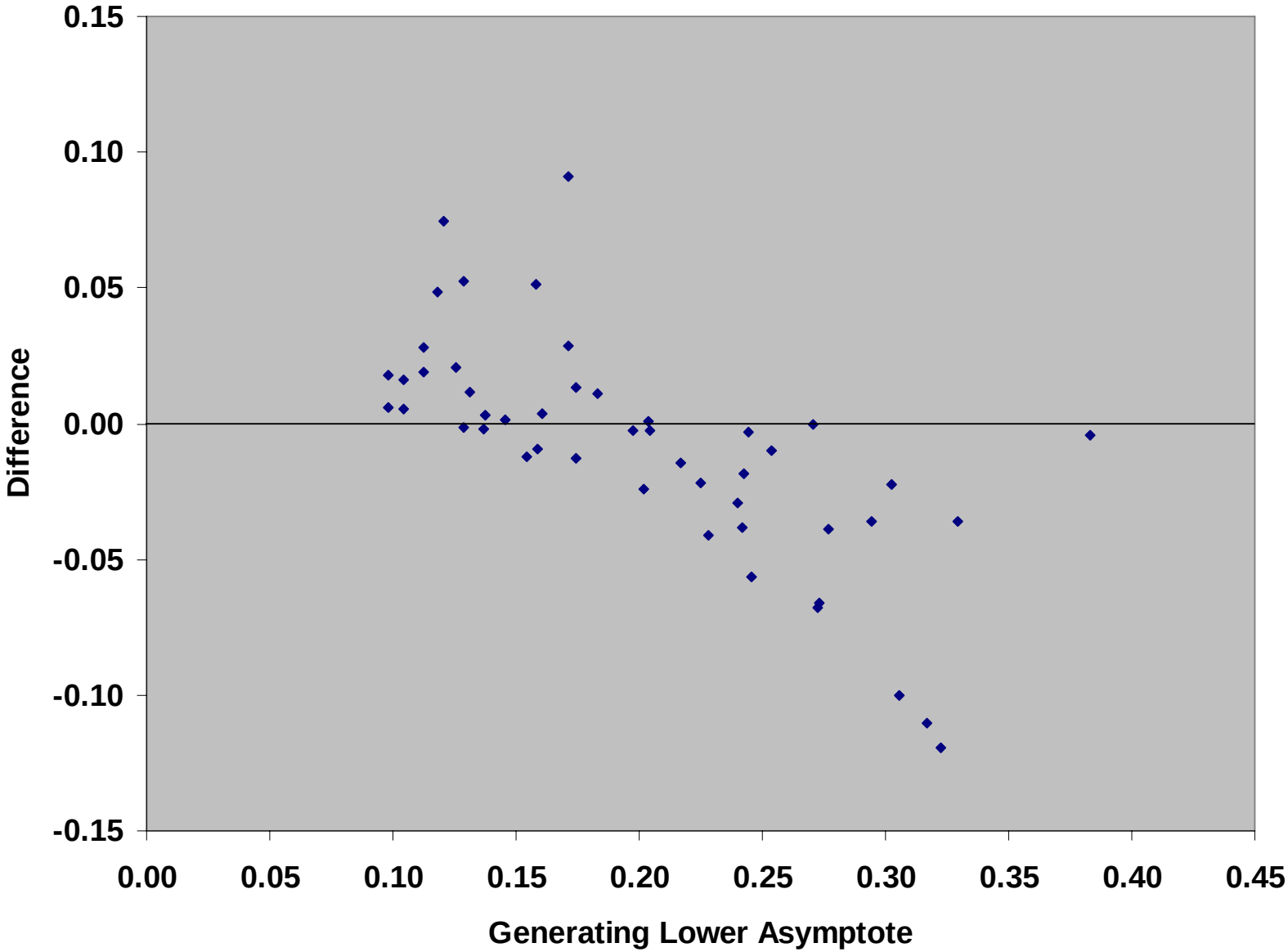
Example 4 – Slopes



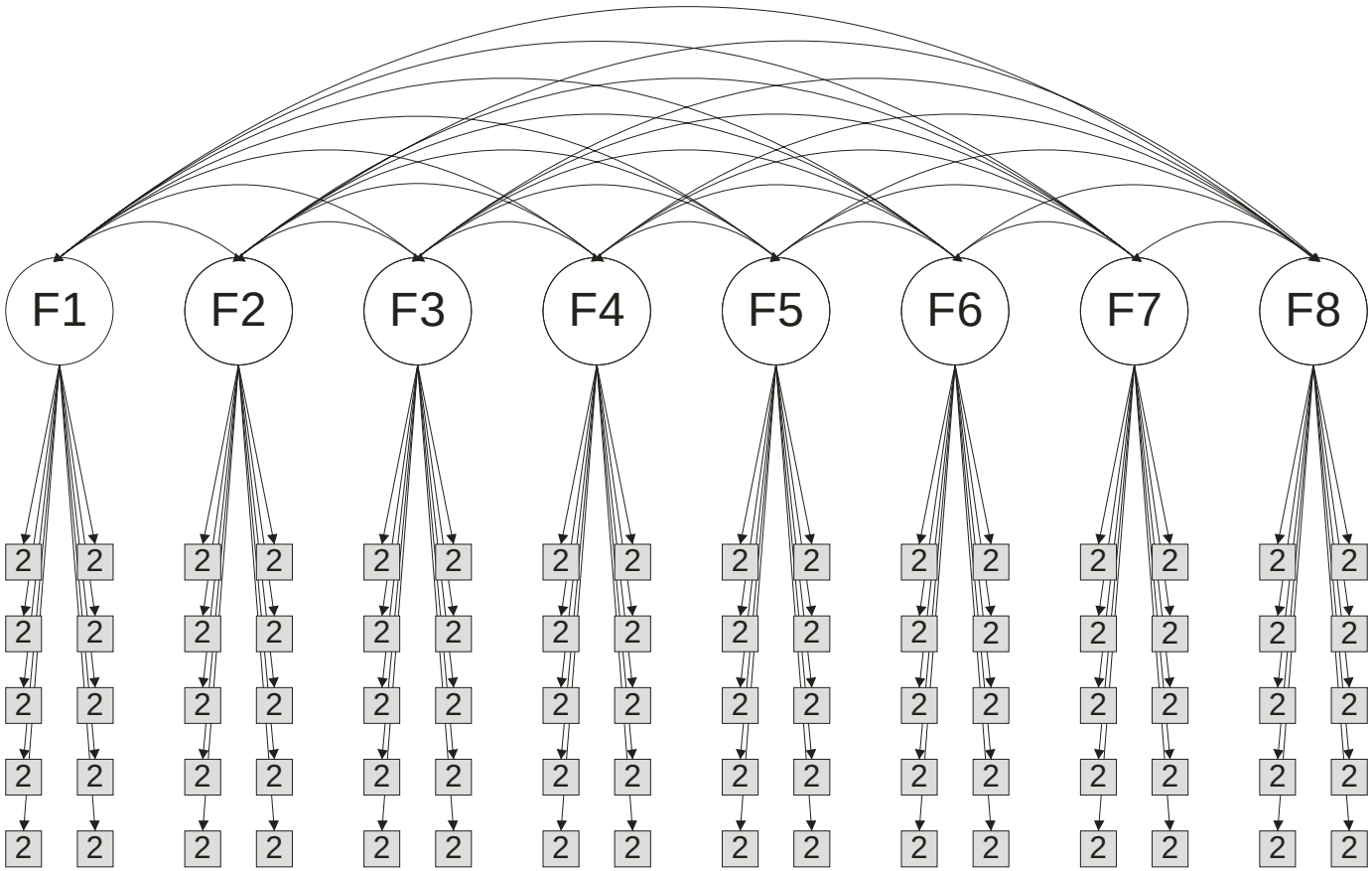
Example 4 – Intercepts



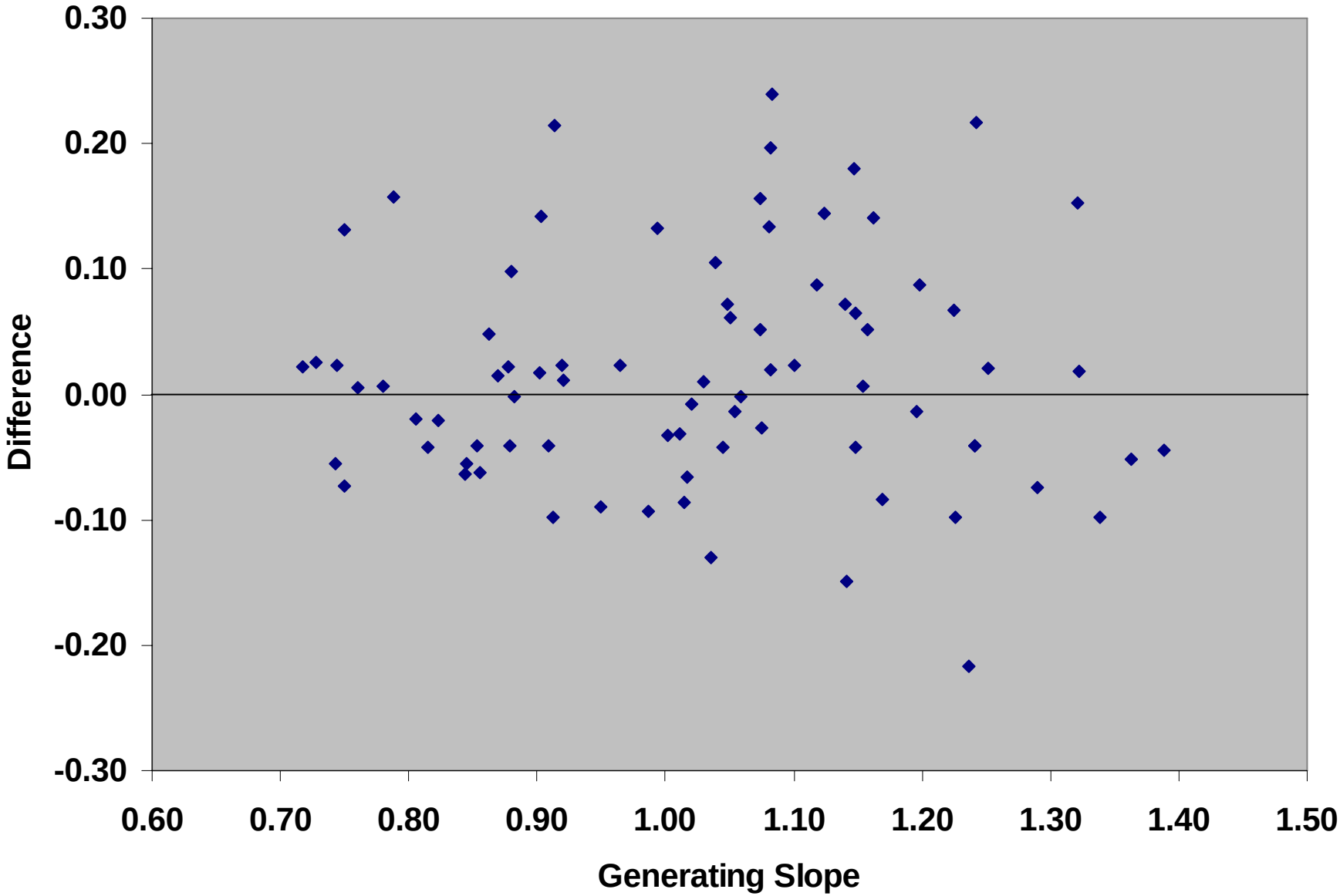
Example 4 – Lower Asymptotes



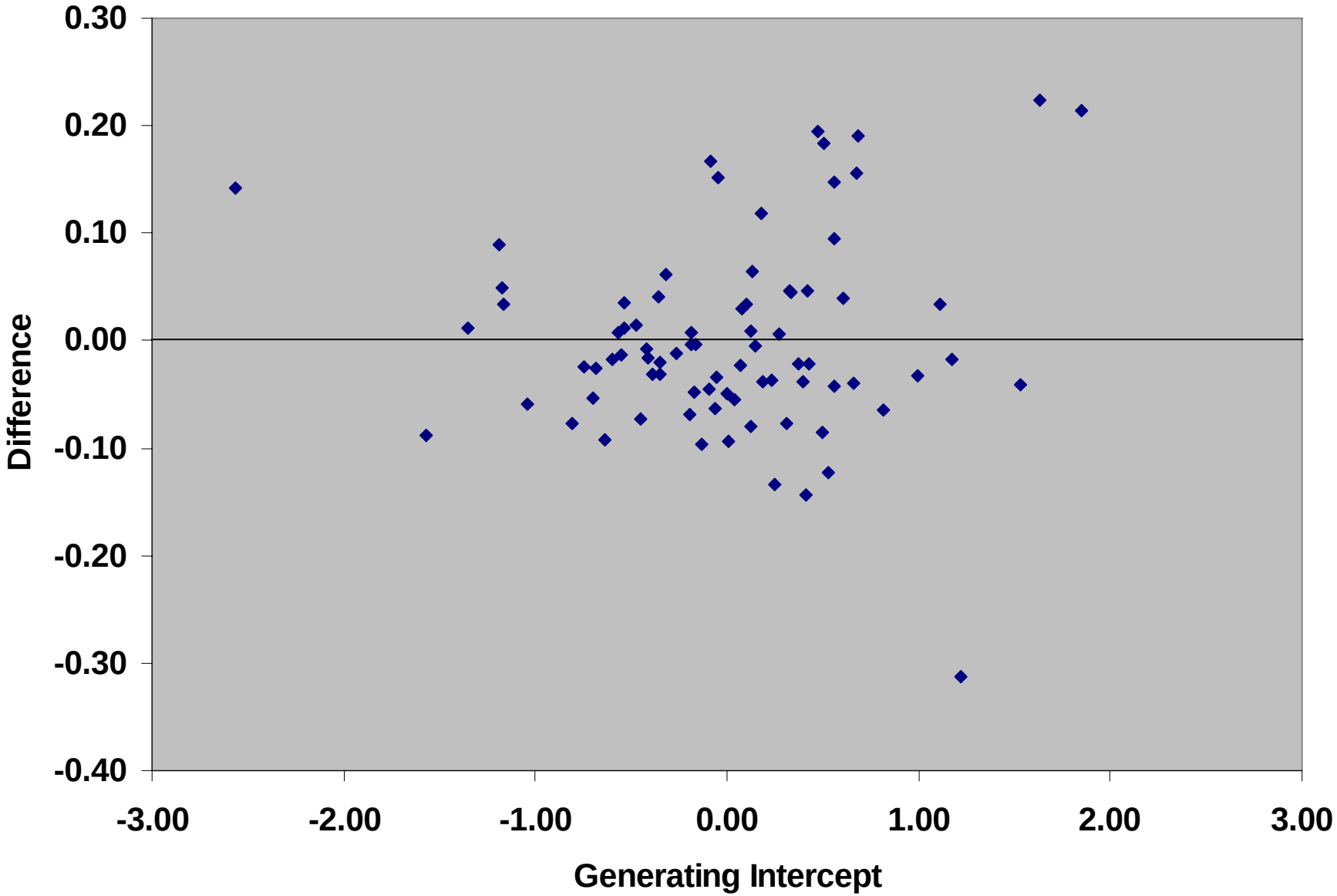
Example 5



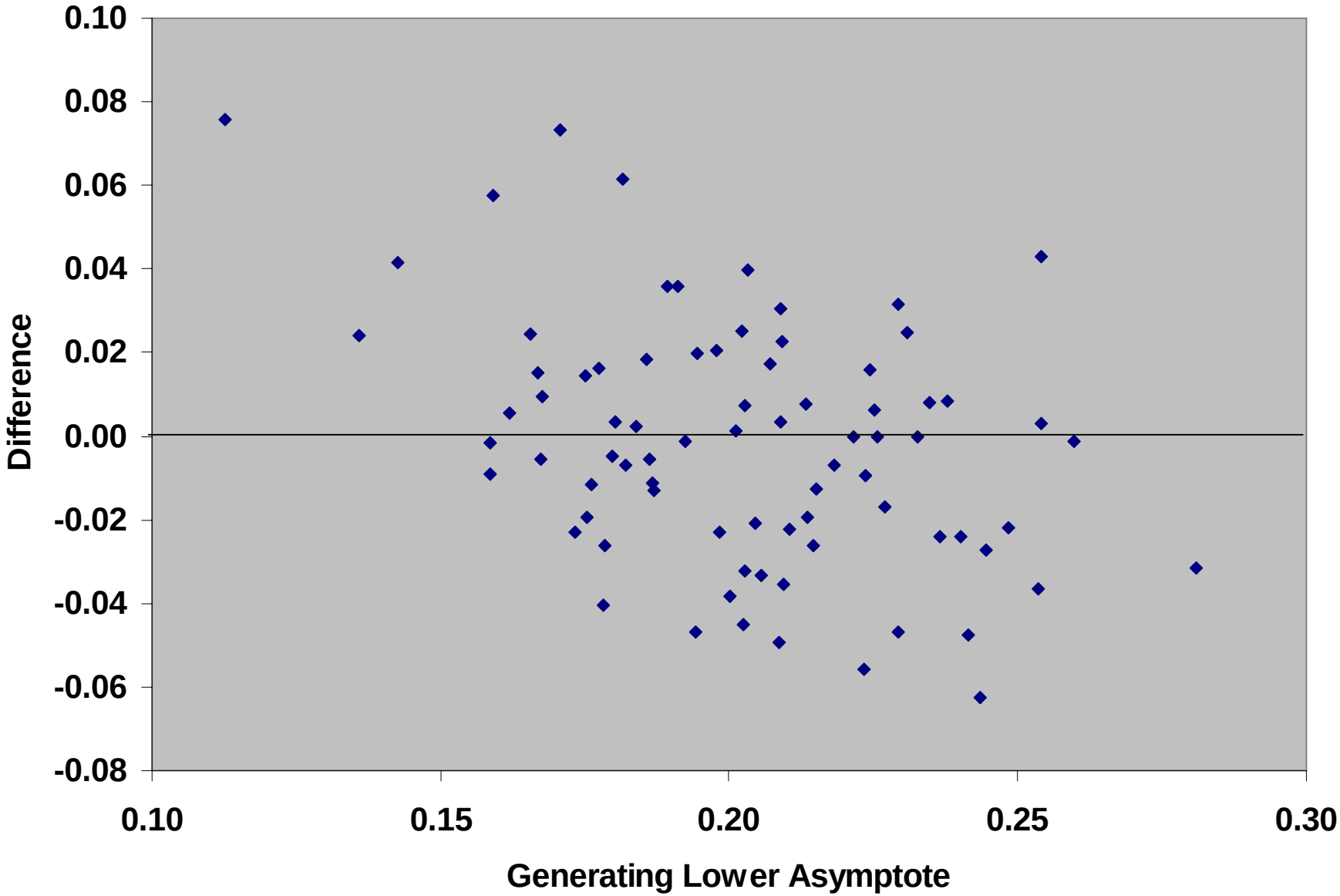
Example 5 – Slopes



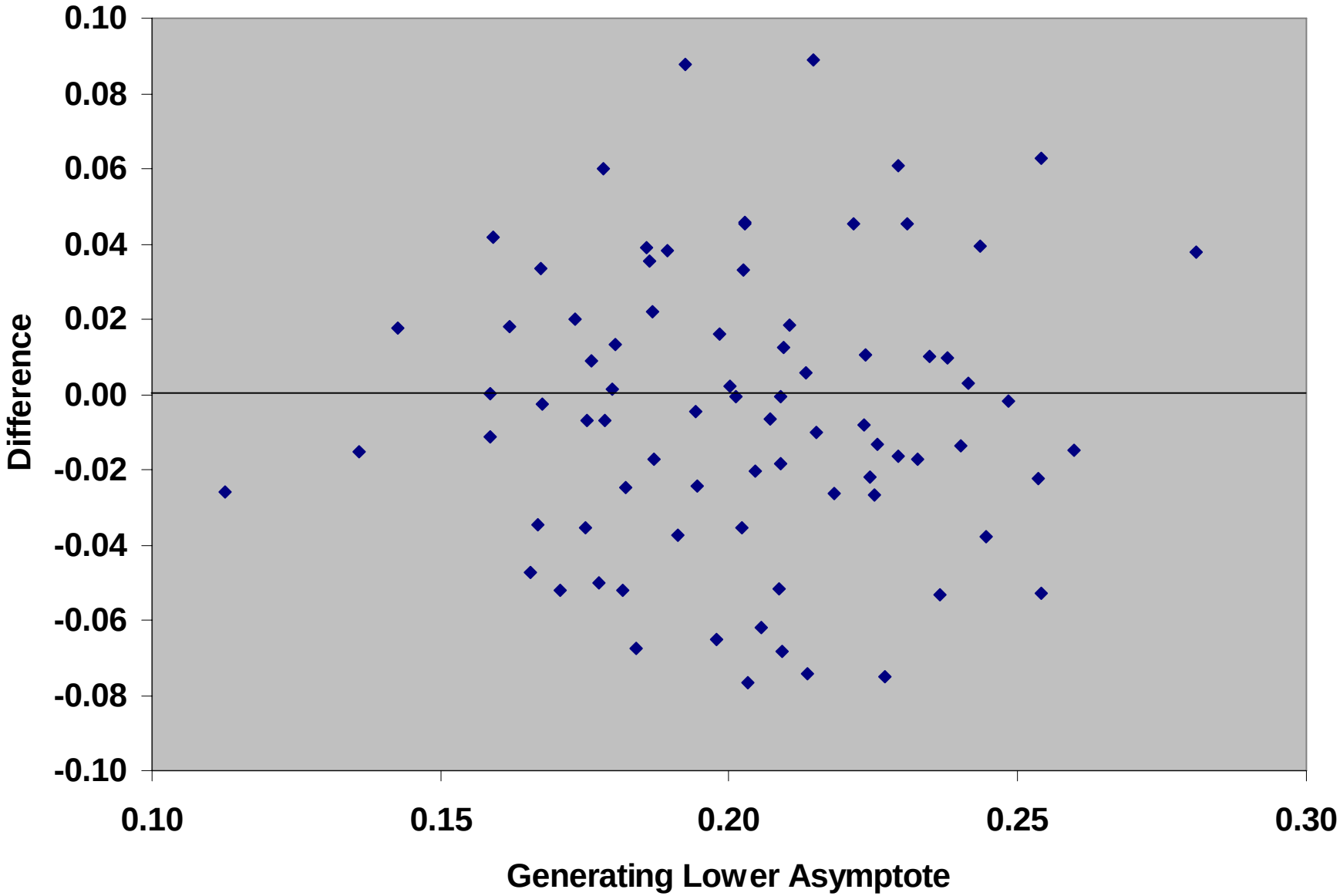
Example 5 – Intercepts



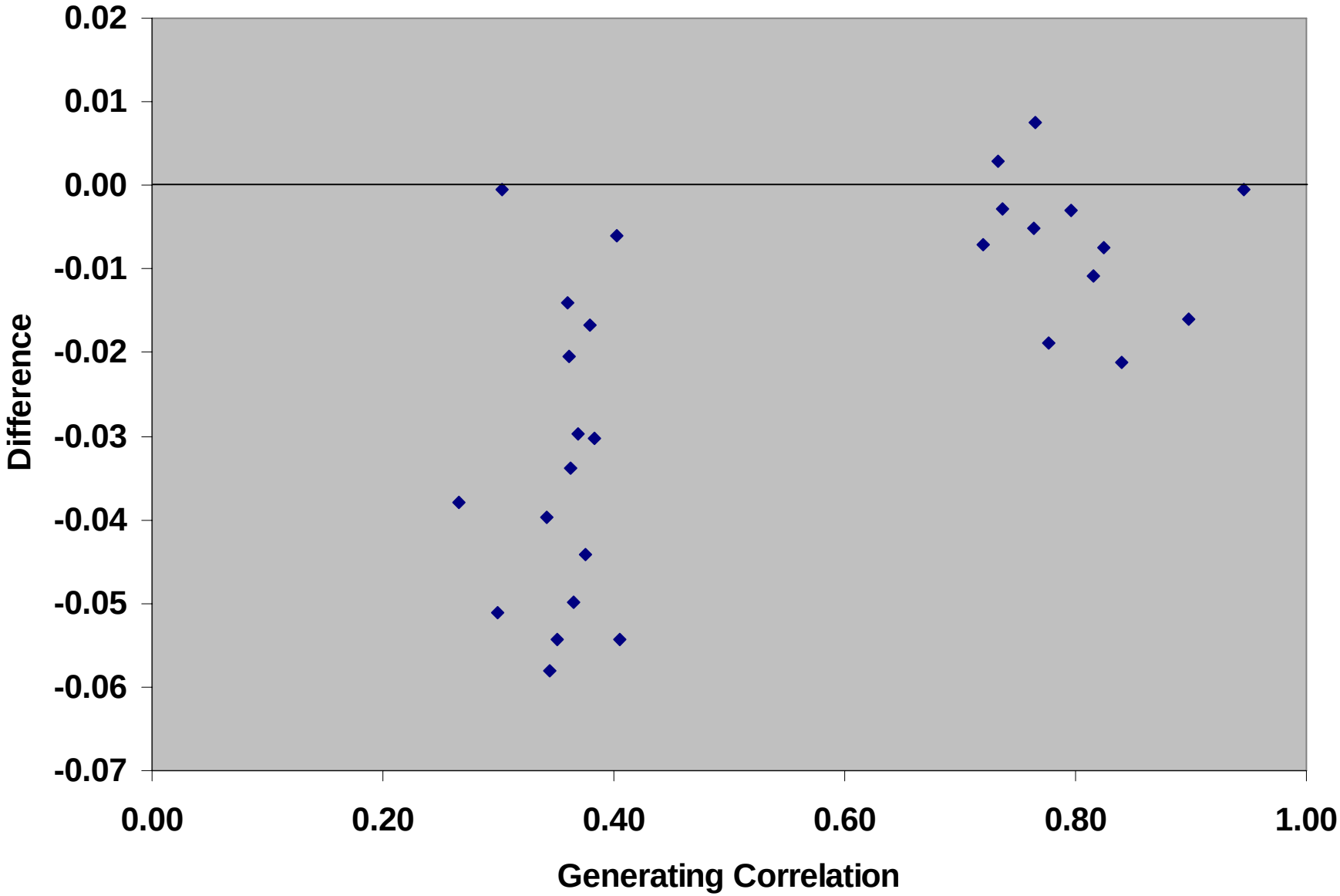
Example 5 – Lower Asymptotes



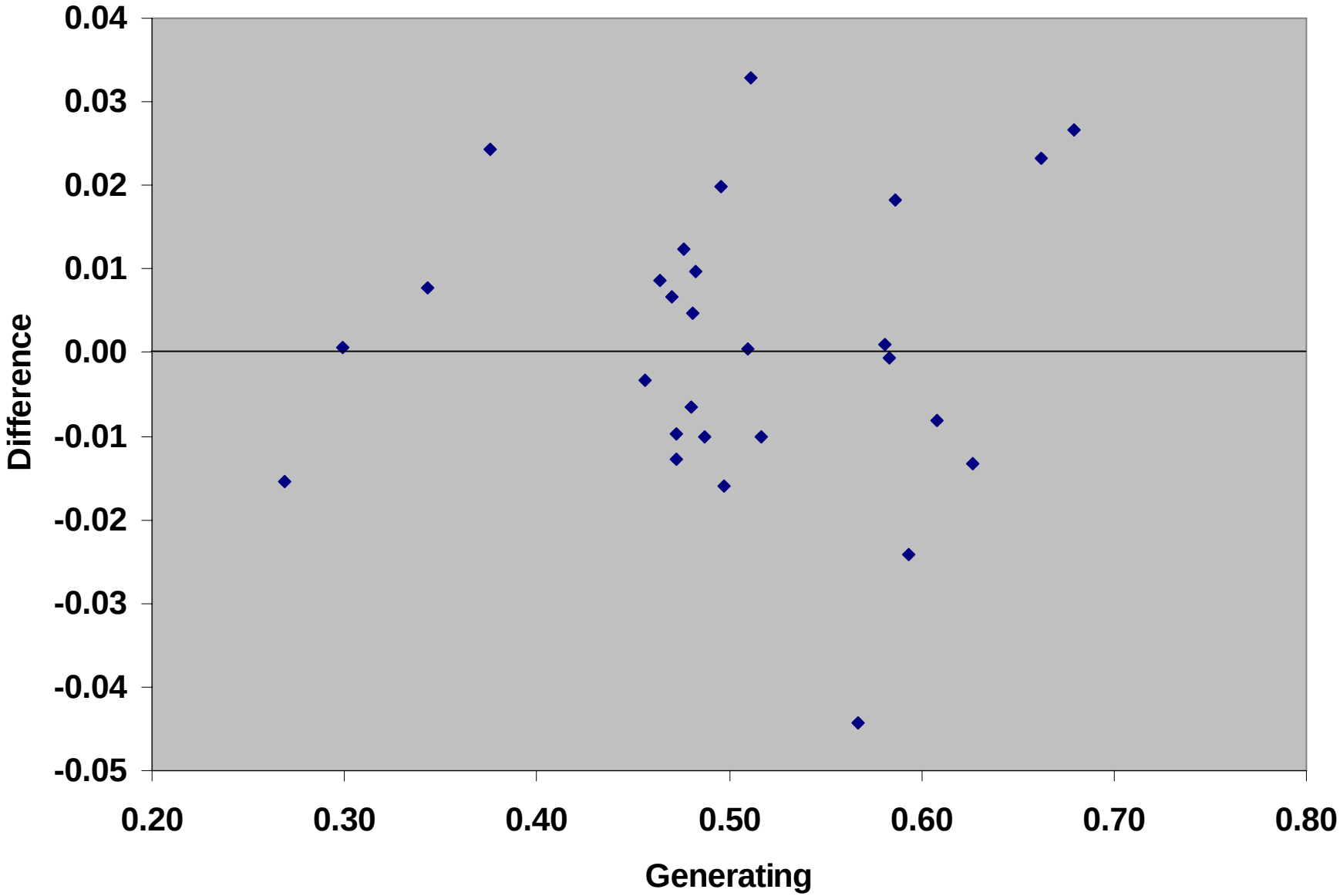
Beta Weight = 20



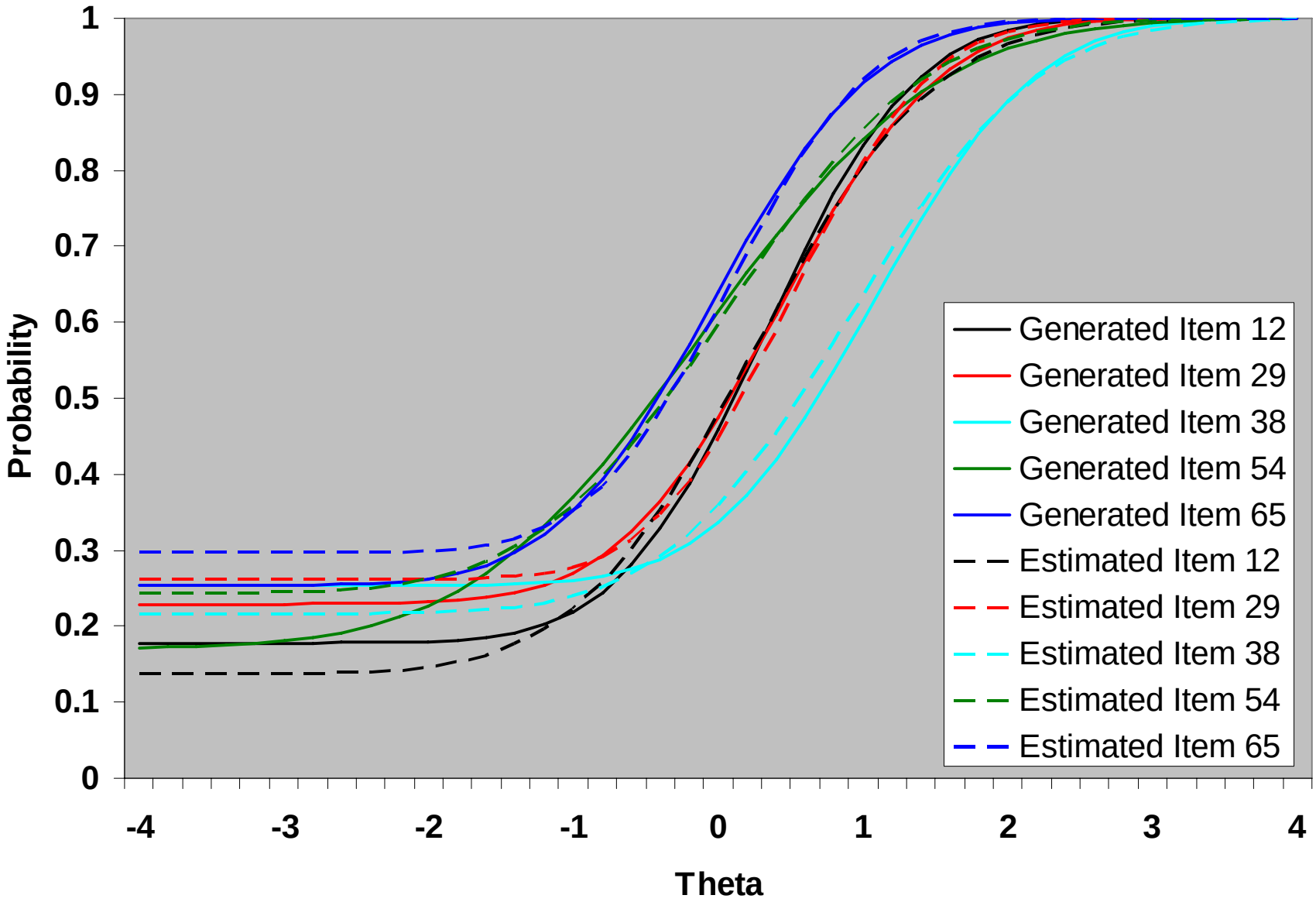
Example 5 – Correlations



E5 Continued: More Uniform R



5 Most Poorly Recovered Items



Future Directions

- Software Dissemination
- Projects
  - MCMC variant comparison
  - Confirmatory item factor analysis and applications
  - MCMC vs. MML-EM, WLS, etc.
- Extensions
  - Structural Model
  - Model Fit

Concluding Thoughts

- Item factor analysis is a useful method for the social sciences
- IRT framework provides an advantageous platform for item factor analysis
- MCMC can be used to estimate parameters for more complex IRT models

Acknowledgements

- Dave Thissen
- Dissertation Committee
  - Ken Bollen, Patrick Curran, Andrea Hussong, & Bud MacCallum
- Jean-Paul Fox
- Psychometric Society (& Judges)

Thank You.

edwards.134@osu.edu

http://faculty.psy.ohio-state.edu/edwards/