first- or second-order isomorphism and are displayed on mental media preserving physical properties (e.g., spatial distances for visuo-spatial representations; Kosslyn et al. 1978). Because the processes acting on analog representations are sensitive to these properties, they can be traced in behavioral performance. Using criteria from seminal papers in cognitive (neuro)psychology, we highlight three important mistakes compromising CK&W’s argumentation.

A first mistake consists in attributing the properties of the content of a representation (representation of object/concept X with properties P) to its format (representation of object/concept X with properties P; Pylyshyn 1978; 1981). Format and content being independent, content properties can be represented in modality-neutral and modality-specific formats (Caramazza et al. 1990). Stating that a numerical representation is abstract if behavioral effects depend only on magnitude is thus erroneous. Contrary to what CK&W write, McCloskey’s semantic representation was not abstract because of its quantitative content, but because of its propositional format. Dehaene’s magnitude representation on an analog medium provides the a contrario argument. Another common mistake concerns the inferences drawn from the (in)dependence between semantic representations and the various modalities/notations of access. Although abstract representations are amodal, a representation accessed through several modalities/notations is not necessarily abstract. Number semantics could be accessed from and give rise to similar effects in verbal, Arabic, or non-symbolic inputs while being non-abstract (e.g., Dehaene’s analog representation). Conversely, modality-notation-specific effects do not necessarily sign non-abstract representations, because abstract representations could be accessed differently by modality-specific presemantic systems (Riddoch et al. 1988). By simply defining non-abstract representations as “sensitive to input modalities” (see sect. 2, para. 1), CK&W fail to provide a constraining framework. For example, finding that, after habituation, magnitude processing in one notation becomes more vulnerable to virtual lesions of the parietal cortex than in other notations, does not contradict the abstract view, as habituation could have modified the connectivity between notation-specific systems and abstract representations prior to the lesion. Trying to consider representations and processes separately constitutes CK&W’s third mistake: Whatever their format, representations cannot be conceived as mental entities distinct from the processes acting upon them (Anderson 1976; Palmer 1978), and processes are totally determined by the format of the representations upon which they act (Shepard & Podgorny 1978). Therefore, any proposal of non-abstract representations must come along with a set of compatible processes clearly specified both at the functional and anatomical levels.

To properly assess the format of numerical representations, we make the following recommendations. First, a data type (a way of organizing information in memory; Simon 1978), and the primitive operations that can be performed on it, must be specified and supported by unambiguous behavioral effects. Rightward biases on the number line in neglect patients (Zorzi et al. 2002) is an instance of how intrinsic properties of representations may show through behavioral effects, but other frequently cited effects are not: The distance effect can be accounted by various factors like “abstract/analogue,” “(a)modal,” “supramodal,” or “modality-(in)dependent,” and so forth, with great care, because each of these conveys a specific meaning. Far from being a rhetorical question, this issue has strong theoretical and empirical implications. It is now time to overcome mistakes that hamper research on numerical representations and their relationship with the functioning of the human brain.

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Numerical representation, math skills, memory, and decision-making

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Ellen Petersa and Alan Castelb

Abstract: The consideration of deliberate versus automatic processing of numeric representations is important to math education, memory for numbers, and decision-making. In this commentary, we address the possible roles for numeric representations in such higher-level cognitive processes. Current evidence is consistent with important roles for both automatic and deliberative processing of the representations.

The consideration of deliberate versus automatic processing of numeric representations is important to both math education and decision-making. Numeric information is ubiquitous in the decisions that we make, and thus numeric representation may play an important role. Basic number skills have only recently received attention in the decision literature. Numeracy, defined as the ability to process mathematical and probabilistic concepts, for example, has been shown to reduce susceptibility to framing effects and improve judgment accuracy (Peters et al. 2006). However, the effects of numeric representations in decisions have been largely ignored with two recent exceptions (Furlong & Opfer 2009; Peters et al. 2008). These lines of research raise important questions concerning relations between numeric representations, math skills, and the use of numeric information in decisions. This commentary adds to the Cohen Kadosh &
Walsh (CK&W) viewpoint by addressing the possible roles of automatic and deliberate processing of numeric representations in higher-level cognitive processing. 

**Numeric representations and math skills.** Peters et al. (2008) tested a large sample of healthy younger and older adults (mean age ≈ 20 and 70 years, respectively) using a distance-effect reaction-time task with Arabic integers and dots (“Is the quantity shown bigger or smaller than 5?”), among other notations. In a reanalysis of the younger-adult data including only these two notations, the size of the distance effect varied by notation, with more precise representations (smaller distance effects) for Arabic integers than dots, supporting CK&W's point that numeric representations are not necessarily abstract. CK&W claim further support from findings that the distance effect with integers relates to math achievement, but the distance effect with dots does not. However, in the reanalysis of Peters et al.'s data, more precise numeric representations for both Arabic integers and dots were associated with higher numeracy scores. Halberda et al. (2008) demonstrated a similar positive relationship between 14-year-old children's performance on a task that used only numerosities (dots) and their math ability back to kindergarten.

The question remains whether this association is based on automatic or deliberate processing of the representations. An age comparison may be illuminating because of the shift that occurs in reliance on more deliberative to more automatic processes from younger to older adulthood across a variety of domains. If the relation between numeracy and numeric representations is based on more automatic processes, then this correlation should increase, as older adults tend to rely relatively more on automatic processes (see Peters et al. 2007 for a review); if it is based on more deliberative processes, one might expect a decreased correlation. In Peters et al.’s (2008) original data, the distance effect was less associated with numeracy for older than younger adults (r = .06, p = .64 and r = .41, p = .01, respectively). These data best support deliberate processes underlying numeracy's association with numeric representations. In addition, Castel (2007) found that older adults who were accountants and bookkeepers demonstrated memory ability remarkably similar to younger adults for arbitrary numbers, but not for arbitrary non-numeric information (where the usual age declines in memory were found). In another study, older adults were exceptionally good at remembering grocery-store prices, but not arbitrary prices (Castel 2005). Such data could support a more precise abstract representation for these individuals (who have more experience with numbers in general or grocery prices in particular) that is carried into later life. However, in combination with Peters et al.’s data, these findings are also consistent with motivated selectivity in deliberative processing (Hess 2000), with some numeric information (or all numbers for some individuals) being particularly important and valued.

**Numeric representations and decision-making.** In accord with CK&W’s notion of a deliberate level of numeric representations, decision-makers are thought to use numeric information intentionally and deliberatively (e.g., stock-market indicators, mortgage rates, and grocery bills). Because human decision-making likely derives in part from the same mechanisms evolved by other animals in response to risky natural environments, intuitive representations of symbolic numbers should relate to how individuals respond to numeric information in decision-making. Peters et al. (2008) developed and tested hypotheses relating an individual-difference measure of the size of the distance effect to decision-making. They hypothesized that individuals with more precise representations would weight proportional differences between numeric attributes in choice more than individuals with less precise representations. In addition, a larger proportional difference should result in a bigger difference between individuals than a smaller proportional difference.

Results of two decision studies in Peters et al. (2008) supported these hypotheses. In the first study, individuals with more precise representations (compared to those with less precise representations) were more likely to choose larger prizes received later than smaller, immediate prizes, particularly with a larger proportional difference between the two monetary outcomes. In a second study, they were more likely to choose a normatively worse option that saved a greater proportion of lives at risk (but a smaller number of lives) compared to those with less precise representations. Importantly, these findings were more consistent with the abstract-representation view, because the results of both studies held after controlling for numeracy and various measures of intelligence associated with prefrontal activity. The precision of number representations appears to underlie: (a) perceived differences between numbers, (b) the extent to which proportional differences are weighed in decisions, and, ultimately, (c) the valuation of decision options. Human decision processes involving numbers important to health and financial matters may be rooted in elementary, biological processes shared with other species, and which depend on an automatic representation of numeric information across notations.

It is critical to better understand number representation in the context of how individuals use numbers. The CK&W article forces us to consider whether the role of these representations in higher-order cognitive processing emerges from a shared representation across notations resulting more from prefrontal activity or whether their role results from an abstract representation. The current results are most consistent with numeracy being related to the former shared representation and decision-making being associated with the latter automatic processing of the representations. Human decision-making, however, often involves prefrontal activity (Rangel et al. 2008), and further consideration of numeric representations being some combination of abstract and deliberate may converge with and ultimately explain some findings in the neuroanatomy of decision-making. It remains plausible that a shared deliberate representation (that is separate from what is associated with numeracy) could explain Peters et al.’s findings. A better understanding of the automatic versus deliberate nature of these representations' influences on decision-making will illuminate the important contribution of numeric representations on everyday decisions and should ultimately lead to improvements in decision aids.

**What is an (abstract) neural representation of quantity?**

Manuela Piazza\(^a\) and Veronique Izard\(^b\)

\(^a\)Center for Mind Brain Sciences, University of Trento, 38058 Rovereto (TN) Italy; \(^b\)Department of Psychology, Harvard University, Cambridge, MA 02138.

Manuela.piazza@unitn.it Veronique.izard@polytechnique.org

Abstract: We argue that Cohen Kadosh & Walsh’s (CK&W’s) definitions of neural coding and of abstract representations are overly shallow, influenced by classical cognitive psychology views of modularity and seriality of information processing, and incompatible with the current knowledge on principles of neural coding. As they stand, the proposed dichotomies are not very useful heuristic tools to guide our research towards a better understanding of the neural computations underlying the processing of numerical quantity in the parietal cortex.

According to Cohen Kadosh & Walsh (CK&W) a neural representation of quantity is abstract if “neuronal populations that code numerical quantity are insensitive to the form of input in which the numerical information was presented” (sect. 2, para. 1). This