Describing perceptual information about human growth in terms of geometric invariants

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Three experiments are reported that examine the perceptual information by which growth of a human head is distinguished from other possible styles of change. The results demonstrate that the perception of growth may be based on a specific set of invariant geometric relations described previously by Mark, Todd, and Shaw (1981). When observers are asked to judge which of two facial profiles looks older, they are surprisingly accurate if the profiles are related by an appropriate transformation, but their performance is no better than chance if the profiles are related by an inappropriate transformation. If, on the other hand, they are instructed to judge whether a pair of profiles is the same or different, the differences produced by the appropriate and inappropriate transformations are equally discriminable.

Human observers encounter many distinct styles of change as they move about in a natural environment. Consider, for example, the possible distortions of a visible surface such as bending, stretching, twisting, or tearing, and the different varieties of bipedal gait, such as running, walking, skipping, or jumping. Each of these events is perceived by human observers as a categorically distinct entity. It is important to keep in mind, moreover, that this particular list of examples is but a small number of the myriad of events observed in nature. The number of perceptually distinct categories of change is extremely large, yet human observers are able to identify these categories with uncanny reliability.

A casual survey of the many different styles of change encountered in our day-to-day experiences reveals two fundamental facts: First, a given category of change can be identified over a wide variety of different objects, including those that are unfamiliar (e.g., a wheel, a log, or a stone can all be perceived as rolling). Second, there are a potentially large number of different categories of change that can be correctly identified for an individual object (e.g., a cube of ice could be perceived as sliding, spinning, or melting). These observations provide strong evidence that the perception of a style of change is largely independent of the particular object to which it is applied. This is, indeed, a fundamental aspect of human event perception that any adequate theory must address.

What are the sources of information that specify a particular style of change in such a broad range of contexts? One possible answer to this question is suggested by the classification of geometries proposed by Felix Klein in his famous Erlanger Program. Klein's method of classification was based on the fact that any geometric transformation will selectively alter some of an object's properties while leaving others unchanged. If, for example, we translate or rotate an object in space, its position will change but its size and shape will remain invariant. If, on the other hand, the object were made of rubber and we deformed it at random, its size and shape would be altered, but the adjacency relationships for neighboring regions of its surface would remain the same. Klein's Erlanger Program was an attempt to discover a hierarchical relationship among different geometries (e.g., Euclidian, affine, projective, etc.) by associating each one with a particular group of transformations, and by analyzing the properties of figures that are preserved under the transformations of each group. (See Shaw and Pittenger, 1977, for a more detailed discussion of Klein's hierarchy of geometries.) A similar method of classification might also be applicable to human perception—that is to say, perhaps the identity of a perceptually distinct style of change is visually specified by the particular constellation of object properties it leaves invariant.

One source of evidence that supports this hypothesis comes from recent investigations of the perception of craniofacial growth. As an individual grows from infancy to adulthood, the shape of the head changes systematically in a way that naive observers can readily identify as resulting from growth, as opposed, for example, to gain-

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ing weight or undergoing plastic surgery (Mark & Todd, 1983; Pittenger & Shaw, 1975; Pittenger, Shaw, & Mark, 1979; Todd, Mark, Shaw, & Pittenger, 1980; Todd & Mark, 1981). Within this overall pattern of change, however, there are certain properties of craniofacial structure (e.g., its bilateral symmetry) that remain quite stable over time. Mark, Todd, and Shaw (1981) have recently investigated several of these geometric invariants and have demonstrated their significance for the perceptual identification of growth events. Using a variety of different mathematical transformations applied to profile drawings of a human head, Mark et al. found that human observers would label a particular profile change as growth only if the mathematical transformation from which it was generated preserved the appropriate set of geometric invariants.

One important issue that was left unresolved in this earlier research concerns the qualitative nature of growth events in relation to other types of change. Is craniofacial growth a perceptually distinct category with tightly defined boundaries? Or is there a continuum of some sort on which any given pattern of change can be said to resemble growth to a greater or lesser extent? The research described in the present article was specifically designed to address this question. In order to assess observers' sensitivity to how closely a given transformation resembles growth, a discrimination task was employed in which pairs of facial profiles were presented and observers were asked to judge the member of each pair that appeared older. A similar procedure was then employed to measure observers' sensitivity to the mere presence of change for the same transformations. The results of these experiments provide strong evidence that craniofacial growth is indeed a well-defined perceptual category and that geometric invariants are a primary source of information from which it is identified.

EXPERIMENT 1

Method

Transformations. The current study uses five transformations from Mark et al.'s (1981) study. These transformations differ with respect to the following geometric invariants: (1) orientation of every point on the profile is maintained with respect to the origin of the coordinate system (note that, in Figure 1, \( \theta = \theta \)); and (2) bilateral symmetry across the vertical axis is preserved. Cardiodial strain and spiral strain maintain both invariants; reflected shear maintains only the latter invariant; affine shear and rotation fail to preserve either invariant (Figure 1). (Revised cardiodial strain was not used either by Mark et al. or in the current study.)

Stimulus sequences. The stimuli for this paired comparison study were taken from the sequences of facial profiles constructed by Mark et al. (1981) (Figure 2). These stimulus sequences were prepared from longitudinal growth records collected by the Child Research Council of Denver, Colorado. One group of stimuli, the actual growth sequences, provided a baseline measure for evaluating the different transformations. Each actual growth sequence consisted of facial profiles of a single individual at five ages, ranging from roughly 6 to 20 years. These profiles were traced directly from X-ray plates made with small amounts of radiation so that the outline of the skin was clearly visible. The overall change in facial angle between the youngest profile and the oldest one was used as an index of the amount of change produced by actual growth. A second group of facial profiles, the transformation sequences, were computed by systematically transforming the youngest profile of each actual growth sequence. Specific values of the free parameter of the cardioidal strain, spiral strain, affine shear, reflected shear, and rotation transformations (Figure 1) were selected for each individual so that the overall change in facial angle would be identical with the change that had occurred as a result of that individual's actual growth. Actual growth sequences for five individuals (hereafter referred to as "patients") were constructed, thereby allowing us to produce five complete sets of sequences similar to that shown in Figure 2. (The reader may consult Mark et al., 1981, for further details of the stimulus preparation.)

Stimulus preparation. For each sequence of five profiles, 10 different pairs of heads can be formed. Each pairing was Xeroxed twice, once with the older profile to the left of the younger profile and once to the right. This resulted in 20 pairings for each of the six sequences (Figure 2). Thus, there were 120 profile pairings for each of the five patients for whom actual growth sequences had been constructed.

The pairs of profiles differed by one to four units, where a "unit" refers to the difference in facial angle between adjacent profiles in any transformation sequence. In any sequence, for example, Profiles 1 and 2 or 3 and 4 differ by one unit, whereas Profiles 1 and 5 differ by four units. Of the 10 different pairings for each sequence, four pairings differed by one unit, three differed by two units, two differed by three units and one differed by four units. The number of pairings at each unit difference was not equated because the results of pilot work had indicated that the various transformations were most likely to be distinguished on comparisons of small profile differences.

Subjects. Forty-five undergraduates participated in the experiment for course credit.

Procedure. Subjects were told that pairs of profiles were arranged on each sheet in their test booklets. Their task was to select the older profile of each pair. The subjects were allowed to work at their own pace. The subjects were divided into five groups with each group viewing all 120 profile pairings that were constructed for a single patient. The order of their presentation was randomized within the constraints that the same pair of profiles appear in different halves of the test and that the older profile appear on the left side an equal number of times in each half of the test.

Scoring procedure. The "correct" response for each pair of transformed profiles was the profile that was produced with the larger value of the free parameter (i.e., the profile with the larger facial angle).

Results and Discussion

Since there was a different number of comparisons at each unit level of difference, the total number of errors for the various transformations and actual growth at each of the four comparison differences was recorded as a proportion—the actual number of errors divided by the number of possible errors. Figure 3 shows the percentage of correct age ("older") judgments, averaged over the five patients, for each transformation and actual growth at the four levels of comparison difference. This figure reveals that the percentage of correct judgments increased monotonically for the four transformations and actual growth as the size of the comparison difference increased. More importantly, Figure 3 shows that, at each level of difference, cardioidal strain and spiral strain were judged more accurately than either affine shear or reflected shear. This result was observed for each of the five patients whose
RIGID ROTATION
(POLAR COORDINATES)

\[ \Theta' = \Theta + k \]
\[ R' = R \]

CARDIOIDAL STRAIN
(POLAR COORDINATES)

\[ \Theta' = \Theta \]
\[ R' = R(1 - k \cos \Theta) \]

SPIRAL STRAIN
(POLAR COORDINATES)

\[ \Theta' = \Theta \]
\[ R' = R(1 + k|\Theta|) \]

REVISED CARDIOIDAL STRAIN
(POLAR COORDINATES)

\[ \Theta' = \Theta \]
\[ R' = R(1 + k(1 - \cos \Theta)) \]

AFFINE SHEAR
(RECTANGULAR COORDINATES)

\[ Y' = Y \]
\[ X' = X + Y \tan \Theta \]

REFLECTED SHEAR
(RECTANGULAR COORDINATES)

\[ Y' = Y \]
\[ X' = X + (Y \tan \Theta) \frac{X}{X'} \]

Figure 1. The shape of an object can be altered within a fixed coordinate system by applying a variety of geometric transformations, six of which are listed by name and also represented in the form of the equations at the left. (The fixed coordinate system is given in parentheses.) The effects of the different transformations on a square are shown at the right.
actual growth data were used to construct the transformed profiles in Figure 2. The percentage of correct comparisons involving the two strain ("growth") transformations were comparable to those for actual growth [cardiodal strain, \(\chi^2(3) = 1.26, p > .50\); spiral strain, \(\chi^2(3) = 1.36, p > .50\)], in contrast to judgments of the transformations that did not preserve the three growth invariants—affine shear \([\chi^2(3) = 14.07, p < .01]\), reflected shear \([\chi^2(3) = 13.03, p < .05]\), or rotation \([\chi^2(3) = 18.32, p < .001]\). This difference is also reflected in the weighted mean percent differences between comparisons involving actual growth and each of the transformations averaged over unit levels of difference—cardiodal strain, 5.1%; spiral strain, 5.0%; affine shear, 16.7%; reflected shear, 16.5%; and rotation, 17.5%.

One possible interpretation of these results is that the two classes of transformations differed quantitatively in regard to their depictions of growth, with the nongrowth transformations being seen as less effective models of growth, but growth nonetheless. There is other evidence, however, to discount this interpretation. After completing the paired comparison experiment, a number of subjects remarked that many pairs of faces did not appear to differ in age. But when forced to choose the older profile, they looked for some physical dimension along which the two profiles differed, assigning one end of that dimension as the older end. For example, on debriefing, 11 subjects mentioned that the slant of the profile seemed to provide a useful cue for determining ordinal age relations in those ambiguous cases. If subjects did utilize such a cue, the weak tendency to see the nongrowth transformations as producing changes in age may well be an artifact of the method used to equate the ranges of the transformations.

This possibility was examined in Experiment 2 using a similar paired comparison procedure, in which dif-
INFORMATION ABOUT GROWTH

Figure 3. The percentage of correct ordinal age judgments in Experiment 1, averaged over patients, for each transformation and actual growth at the four levels of difference between the profiles.

Differences in the slant of the pairs of profiles were eliminated as a simple cue for age differences. If subjects in the first experiment were relying on facial slant as a cue for age level, then (in the absence of other reliable cues) we would expect ordinal age judgments to approach chance (50%) on those transformations for which subjects utilize that cue. To provide a baseline for evaluating performance on the reoriented profiles, subjects were also asked to make ordinal age judgments on the same stimuli used in the original paired comparison task, in effect replicating Experiment 1.

EXPERIMENT 2

Method

Subjects. Ten undergraduates participated in this experiment for course credit.

Stimulus preparation. Two sets of profile pairs were used. One set was the original profile pairs from two of the "patients" used in Experiment 1. Those profiles had been oriented so that the Frankfurt horizontal, a line connecting the top of the ear hole and the bottom of the eye socket, was set horizontally on the page. For these pairs, as noted above, the slant of the profiles differed with respect to the horizontal. The reoriented profiles were produced by setting a line connecting the depression above the nose and the outermost protrusion of the chin, to coincide with the vertical. This method for equating orientation turned the rotated profiles into identical profiles. The "correct" profile was, therefore, randomly chosen for each rotation pair. Performance on these rotation pairs should not deviate significantly from chance. Slides were made of all profile pairs.

Procedure. The procedure was nearly identical to that followed in Experiment 1. The two sets of stimuli were shown to 20 sub-

Results and Discussion

Figures 4 and 5 show percent correct judgments for pairs of faces in which profile slant varied and those in which it did not, respectively. For the actual growth, cardioidal strain, and spiral strain transformations, differences in profile slant contributed little to age judgments, whereas for the other classes of transformations, it was a major factor. In addition, a three-factor analysis of variance (with transformation and unit difference as within-subject factors and patient as a between-subject factor) was performed on each task, using the percentage of correct ordinal age judgments.

An analysis of variance performed on subjects' judgments of the profiles in their original orientation (i.e., used in Experiment 1) reveals only two significant effects. As expected, unit difference is significant \[ F(3,24) = 18.34, \] \[ p < .001 \], but more importantly, the main effect of transformation is also highly significant \[ F(5,40) = 19.96, \] \[ p < .001 \]. Figure 4 clearly shows that the latter effect results from a higher percentage of correct judgments of the growth-related transformations in ordinal age transformations (actual growth, cardioidal strain, and spiral strain) than in the nongrowth-related transformations (affine transformations).
shear, reflected shear, and rotation). However, within the nongrowth transformations, subjects' performance is still above chance at each unit difference, and performance increases with unit difference, thus replicating the essential pattern of results from Experiment 1.

A second analysis of variance for the observers' judgments of the reoriented profiles revealed two significant main effects—unit difference \( F(3,24) = 8.89, p < .001 \) and transformation \( F(5,40) = 76.62, p < .001 \). In addition, the interaction between transformation and unit difference is significant \( F(15,120) = 4.80, p < .001 \). Inspection of Figure 5 suggests that the interaction is largely due to the first three levels of unit difference for the nongrowth transformations. Performance did not fluctuate significantly above chance, nor did it increase monotonically with unit difference. Thus, in contrast to the stimuli from Experiment 1, judgments of the reoriented profiles produced by the nongrowth transformations is near chance on all but the largest unit difference.

The effect of profile reorientation was also assessed using a third analysis of variance, in which the dependent measure was the difference in performance on the two sets of stimuli (cf. Figures 4 and 5). This revealed significant main effects of unit difference \( F(3,24) = 7.41, p < .002 \) and, more importantly, transformation \( F(5,40) = 13.44, p < .001 \). None of the interactions was significant. Inspection of Figures 4 and 5 shows reorientation had little effect on subjects' ordinal age judgments of profiles produced by the growth transformations. The mean performance decrement for actual growth profiles is 1.8%; for cardioidal strain, 0.7%; and for spiral strain, 1.7%. By contrast, reorientation produces a marked decrement in the accuracy of subjects' age judgments for the nongrowth transformations: affine shear, 19.1%; reflected shear, 23.2%; rotation, 19.4%.

These results demonstrate that profile orientation has a differential effect on ordinal age judgments for the two classes of transformations. It seems reasonable that in Experiment 1, subjects whose performance was above chance on the nongrowth transformations were using a cue that is the result of an arbitrary method for orienting the profiles and not a product of the transformation per se.

It is important to keep in mind, when evaluating the results of this experiment, that the different transformations perceived as growth can all be distinguished from those that are not by the structural properties of an object they leave invariant (Figure 1). One possible conclusion that can be drawn from this finding is that the invariant properties of a depicted change are a primary source of information for determining its perceptual identity as a growth event. If this conclusion is correct, however, then the differential perceptual effects of these transformations should be context-specific. That is to say, for some other task that has no relation to growth, such as detecting the mere presence or absence of change, the perceptual effects of the different transformations should all be equivalent. Experiment 3 was designed to test this prediction.

### EXPERIMENT 3

**Method**

**Stimuli.** The profile pairings from the paired comparison task (Experiment 1) were used as the nonidentical pairs in the current experiment with the exception of the profiles produced using rotation. Rotation was omitted from this experiment to reduce the number of profile pairs judged by each subject. For each of the five patients, 100 identical pairs of identical profiles were constructed; each of the five profiles from the actual growth and each of the four transformation series was paired with itself four times, thus making equal numbers of same and different pairings.

**Subjects.** Ten undergraduates participated in the experiment for course credit.

**Procedure.** Each subject saw all 200 profile pairings generated for one of the five patients. Slides of the profile pairings were presented using a projector tachistoscope. Each pairing was preceded by a readying cue, "+", that was displayed for 1 sec; after a 1-sec delay, the profile pairing was displayed for 2 sec, the screen was erased, and the subject had to indicate whether the two profiles were physically identical or different. The subjects were told that half of the pairs would be identical. Pilot work had indicated that a 2-sec exposure would produce a roughly 20% error rate, thereby avoiding ceiling effects. The subjects were run in four blocks of 50 trials, with a short pause between blocks.

**Results**

There are two types of errors—false alarms (responding "different" when the profiles are actually the same) and misses (responding "same" when the profiles are ac-
<table>
<thead>
<tr>
<th>Transformation</th>
<th>Unit Difference</th>
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<tr>
<td></td>
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<tr>
<td>Actual Growth</td>
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<tr>
<td>Cardoidal Strain</td>
<td>19.7</td>
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<td>Spiral Strain</td>
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<td>Affine Shear</td>
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<tr>
<td>Reflected Shear</td>
<td>20.8</td>
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Note—Unit difference = 0 are "false-alarm" errors. Errors under unit differences 1-4 are "misses."

The research described in the present article was designed to examine the importance of geometric invariants for the perceptual identification of growth events. Five different mathematical transformations were compared with the effects of actual growth using a forced-choice discrimination task. In two of the experiments, observers were required to judge which member of a pair of profiles appeared older. The results in that case showed clear differences in the perceptual effects of the different transformations. In a third experiment, however, in which observers judged the mere presence or absence of change, all of the transformations were equally salient.

The particular transformations that were or were not perceived as growth can be distinguished from one another by the structural properties of an object they leave unchanged. Note in Figure 1, for example, that the cardioidal strain and spiral strain transformations preserve two important invariants—the bilateral symmetry of an object and the orientation of every point with respect to the origin, both of which are also characteristic of actual growth (see Todd & Mark, 1981). For the affine shear, reflective shear, and rotation transformations, in contrast, at least one of these structural properties of an object is destroyed. The perceptual distinction between these two groups of transformations is revealed most clearly by the results of Experiment 2, in which the different profiles were presented in random orientations. Under those conditions, the accuracy of the age level judgments for the affine shear, reflective shear, and rotation transformations were no greater than chance, even for the largest possible difference between a pair of profiles. With an equivalent unit difference for the actual growth, cardioidal strain, or spiral strain transformations, the observers could detect the older of two profiles with almost 100% accuracy. All of this suggests that craniofacial growth is a well-defined perceptual category, and that geometric invariants are a primary source of information from which it is identified.

An additional source of evidence for the categorical nature of event perception in this context is revealed by the fact that there were almost no differences at all between the actual growth, cardioidal strain, and spiral strain transformations. Apparently, the specific details of these transformations are irrelevant to their perceptual identity as long as the appropriate geometric invariants are preserved. This also suggests, moreover, that there may be many other as yet uninvestigated transformations that could provide "perceptually equivalent" information about the process of human growth. One of these transformations, revised cardioidal strain (Figure 1), has been used by Todd and Mark (1981) to make accurate predictions of the global remodeling of the craniofacial skeleton that results from growth.

It is important to keep in mind when evaluating the results of these experiments that the concept of a geometric invariant is quite general, and is applicable to many other perceptual categories of change in addition to craniofacial growth. Indeed, the classification of transformations by the properties of objects they leave invariant is one of the cornerstones of modern mathematics. It should not be surprising that such a fundamental distinction among different patterns of change should also play a role in human event perception.

Geometric invariants are a particularly useful source of information when the pattern of change to be identified can be described by a single coordinate transformation.
This includes many of the most common events observed in nature, such as translation, rotation, bending, stretching, twisting, or flowing. The usefulness of this type of analysis is less apparent, however, for other commonly observed events, such as a human gait (e.g., Cutting, Proffitt, & Kozlowski, 1978; Johansson, 1973; Todd, 1983) or a pattern of social interaction (e.g., Bassili, 1976; Heider & Simmel, 1944), which cannot be described by a single coordinate transformation. Whether or not these more complex events can also be identified by the properties of objects they leave invariant remains to be demonstrated by future research.

REFERENCES


NOTES

1. A transformation group is a set of transformations that satisfies four requirements: (1) If two transformations are in the set, then their product is also in the set; (2) the product of any three transformations is associative; (3) the set contains an identity transformation; and (4) every transformation in the set has an inverse transformation that is a member of the set.

2. The facial angle is defined by the intersection of two lines. One line, the Frankfurt horizontal, passes through the top of the earhole and the bottom of the eye socket. The other connects the most prominent part of the chin with the deepest part of the depression just above the nose (cf. Mark et al., 1981, their Figure 3).

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