Journal of Experimental Psychology:
Human Perception and Performance
1981, Vol. 7, No. 4, 855-868

0096-1523/81/0704-0855$00.75

Perception of Growth: A Geometric Analysis of How Different Styles of Change Are Distinguished

Leonard S. Mark, James T. Todd, and Robert E. Shaw
University of Connecticut

Although there have been many demonstrations that human observers can accurately recognize a variety of styles of change, such as rolling, walking, or growing, there are no existing theories capable of explaining how one style of change is distinguished from another. The present article offers a hypothesis that any recognizable style of change is uniquely specified by geometric invariants—the abstract properties of a visual display that are preserved by the change. In an effort to provide an empirical test of this hypothesis, several experiments involving the perception of growth were performed. Observers were required to make perceptual judgments of sequences of facial profiles, each of which was constructed by using a different mathematical transformation. The same pattern of results was obtained on both a free response task and a growth rating task: All transformations that were consistently identified as growth preserved the same geometric invariants.

Although there have been many demonstrations that human observers are able to recognize particular styles of change under minimal or unusual viewing conditions (e.g., Cutting, Proffitt, & Kozlowski, 1978; Gibson & Gibson, 1957; Heider & Simmel, 1944; Johansson, 1975; see Johansson, 1980, for a review of this research), there has been relatively little research on the specific properties of visual displays that make one style of change distinct from another (Mark, Todd, Shaw, & Pittenger, in press). This is a major deficiency of existing theories of visual perception. Given the large number of different styles of change that observers can recognize, it is difficult to imagine that the visual system handles each possible distinction as an independent problem. Surely there must be some common framework in which all recognizable styles of change can be distinguished from one another with the application of a few basic principles.

One way of partitioning the set of all possible styles of change into distinct categories is using the concept of a *transformation group*, as first suggested by the German mathematician Felix Klein. In a speech at Erlangen University in 1872, Klein suggested that different styles of change can be represented mathematically by groups of transformations and that different groups can be distinguished from one another by the properties of objects they leave invariant.[1] A given group of transformations allows variation along some dimensions but not others. For example, if we move a right triangle from one place to another, satisfying the requirements of rigid motion, we find that certain geometric properties change but that others do not; its position changes, but its size and shape remain invariant. If, on the other hand, we expand the triangle in a uniform manner, its size changes, but its shape remains invariant. If we place the tri-

This article is based on the doctoral dissertation of L. S. Mark and was supported by grants from NIDR to R. E. Shaw (DE-1RO-DE04990-02) and the University of Connecticut Research Foundation. The authors thank Reuben Baron, William Mace, John Pittenger, and Michael Turvey for their suggestions and criticisms throughout the course of this work, and Sam Weinstein for allowing us to use the longitudinal growth records at the University of Connecticut Health Center.

Requests for reprints should be sent to L. S. Mark, Psychology Department, U-20, University of Connecticut, Storrs, Connecticut 06268.

[1] A transformation group is a set of transformations that satisfies four specific requirements: (a) If two transformations are in the set, then their product is also in the set; (b) the product of any three transformations is associative; (c) the set contains an identity transformation; and (d) every transformation in the set has an inverse transformation that is a member of the set.

angle non-edgewise in front of a light source to project a shadow on a planar surface, the resulting image is still a triangle, but it need not necessarily be a right triangle. If we project the shadow onto a curved surface, we preserve the property that each line segment covers the shortest distance between two vertices, but the sum of the angles is no longer equal to 180°. If we stretch a rubber triangle between our fingers, the property of being a continuous closed contour remains invariant, but all its other geometric properties are altered; the boundedness of the figure can then be destroyed by cutting it with a pair of scissors.

These examples suggest that a given style of change can be uniquely identified by two sets of properties: the properties of objects that are systematically altered by the change and the properties of objects that remain invariant. Group theory, therefore, provides a global framework for distinguishing different styles of change.[2]

Let us now consider a specific example of how group theory can be applied to the perception of change. In a recent series of experiments initiated by Shaw (Shaw, McIntyre, & Mace, 1974), it was demonstrated repeatedly that there is a group of transformations called cardioidal strain, which observers perceive as "growth" when applied to a variety of objects, including human facial profiles (Pittenger & Shaw, 1975), cartoon drawings of birds, dogs, and monkeys (Pittenger, Shaw, & Mark, 1979), or cartoon drawings of inanimate objects such as Volkswagen "beetles" (Pittenger et al., 1979). There are at least three geometric invariants that are characteristic of cardioidal strain (see Figure 1): (a) The angular coordinate of every point on an object is preserved within a polar coordinate system, (b) bilateral symmetry across the vertical axis is preserved, and (c) the continuity of all contours and their direction of curvature are preserved except along the vertical axis. One important fact that has been overlooked in previous investigations is that there are many groups of transformations other than cardioidal strain which satisfy these invariants. Following a suggestion by Gibson (1950), let us assume that such invariants are the basis for perceptual information.[3] Hence, it is reasonable to expect that if two transformations satisfy the same invariants, then they ought to be perceived as similar styles of change. The research described in the present study examines this hypothesis. Evidence is presented that different transformations that preserve the same invariants as cardioidal strain are perceived as growth by naive observers in a variety of contexts whereas other transformations that violate those invariants are rarely perceived as growth.

The present study compares cardioidal strain with three other prospective growth transformations: spiral strain, affine shear, and reflected shear (Figure 1). These transformations were chosen because each produces one or more craniofacial changes that are commonly observed during actual growth (e.g., change in facial proportions or facial angle; cf. Mark, 1979; Todd, Mark, Shaw, & Pittenger, 1980). Like cardioidal strain, the spiral strain transformation preserves the angular coordinate of each point, bilateral symmetry across the vertical axis and continuity of the profile contour; affine shear does not preserve either the angular coordinate or bilateral symmetry, though it does maintain profile continuity; and reflected shear maintains those invariants listed for

[2] The problem for perceptual theorists is to discover the specific variants and invariants that are perceptually salient to human observers. Unfortunately, this may not be an easy task: There are an infinite number of possible transformation groups because there are an infinite number of object properties that could potentially be affected by change; and to make matters worse, an adequate theory of the perception of change cannot be based exclusively on the particular group structures that have already been studied by mathematicians and physicists. There are no known groups, for example, which can adequately distinguish among easily recognized styles of nonrigid motion, such as animate gaits or facial movements. A precise delineation of these perceptually salient styles of change is unlikely to be forthcoming unless the problem is addressed by perceptual psychologists.

[3] For historical accuracy it should be noted that Gibson had not yet developed his concept of perceptual information in the 1950 book. However, he had recognized by this time the important role group theory might play in perceptual theory, since he cited the seminal paper by Cassirer (1944) on this topic.

spiral strain and cardioidal strain, but it does not preserve the radial coordinate of each point.

Since cardioidal strain and spiral strain preserve the same invariants, then we might expect that subjects would perceive both transformations as a single style of change that is categorically distinct from transformations that are not members of that group, such as affine shear or reflected shear.

These experiments also constitute a meaningful extension of previous efforts to validate cardioidal strain as an abstract specification of growth (Pittenger & Shaw, 1975; Pittenger et al., 1979; Shaw & Pittenger, 1977). Although the findings of earlier experiments have revealed that cardioidal strain is perceived as growth more often than is affine shear, a proper comparison of carioidal strain with actual growth is required to demonstrate that the two are perceptually equivalent. In addition, previous work has used either a relative age judgment task or a paired comparison task, procedures that

CARDIOIDAL STRAIN
(Polar Coordinates)*

$\theta' = \theta$
$R' = R\,(1 - k \cos \theta)$

k - free parameter

SPIRAL STRAIN
(Polar Coordinates)*

$\theta' = \theta$
$R' = R\,(1 + k\,|\theta|)$

k - free parameter

AFFINE SHEAR
(Rectangular Coordinates)

$Y' = Y$
$X' = X + Y \tan \theta$

θ - free parameter

REFLECTED SHEAR
(Rectangular Coordinates)

$Y' = Y$
$X' = X + (X/|X|)\,Y \tan \theta$

θ - free parameter

ROTATION
(Polar Coordinates)

$\theta' = \theta + b$
$R' = R$

b - free parameter

*Polar North = 0 degrees

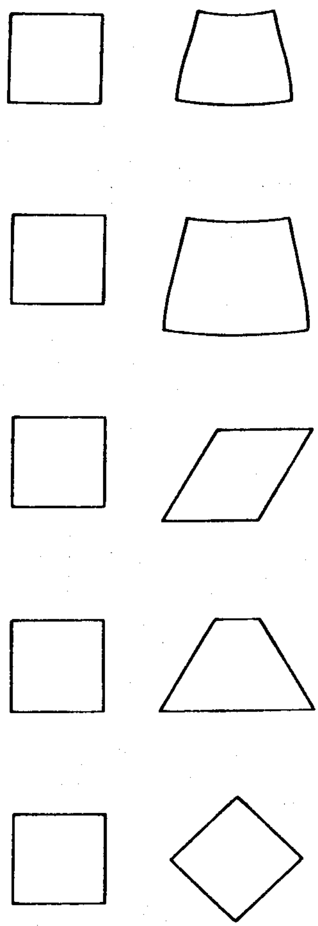

Figure 1. The four prospective growth transformations examined and an illustration of the effects of each transformation.

instruct subjects to try to see the transformed profiles as differing in age. Experiments 1 and 2 of the present study employ free response tasks in an effort to obtain a measure of the *natural salience* of each prospective growth transformation and actual growth. If observers must be prompted in order to see a transformation as producing an increase in age-level, then it could be argued that the transformation is not perceived as effecting that style of change under natural conditions.

Experiment 1: The Uninformed Free Response Task

The first experiment was designed to compare the natural salience of the four prospective growth transformations, described in Figure 1, to actual growth. Subjects, who were unaware of the experiment's relevance to growth or aging, were asked to identify the style of change in sequences of five profiles where any given sequence had been produced by one of the candidate transformations or by actual growth. (See Figure 2 for examples of these sequences.) If one of the transformations is perceptually equivalent to actual craniofacial growth, then it should be perceived as growth about as often as actual growth is and more frequently than any of the other transformations.

Method

Construction of the transformed and actual growth sequences. The perceptual consequences of the four prospective growth transformations and actual craniofacial growth can be measured fairly and reliably only if comparable sequences of transformed profiles are constructed. Each transformation has a free parameter designated by the variable, k, which controls the magnitude of the resultant change. In order to equate the transformations, both with each other and with actual growth, the range of values assigned to the free parameter must produce equivalent physical effects along a profile dimension that changes as a result of growth. The dimension chosen for this purpose was a measure of "facial angle," defined by the intersection of two lines—the Frankfurt horizontal, which passes through the top orb of the ear hole and the bottom orb of the eye socket, and a line connecting the most prominent part of the chin and the deepest part of the depression just above the nose (Figure 3); the facial angle has been commonly observed to increase as a consequence of the global remodeling of the craniofacial complex due to growth.

The actual growth sequences used in the present set of experiments were obtained from longitudinal growth records from the Denver Research Council's Longitudinal Growth Study (1925–1970). (McCammon, 1970). From the longitudinal growth records, the actual growth sequences—all happened to be of males—were constructed: The lateral headplates (X-rays) taken of a individual were examined for the presence of soft tissue profiles. (A soft tissue outline appears on a headplate if it was taken with a low amount of radiation. Only some headplates, however, showed any soft tissue.) An individual was chosen for this study when five properly standardized soft tissue profiles were found that were distributed throughout the age range of 5–23 years, with no more than one profile above the age of 18. Although these individuals are referred to as "patients," they are not known to have any noticeable anomaly. The age and facial angle of the five profiles of each patient in the five actual growth sequences are given in Table 1.

The four candidate transformations shown in Figure 1, cardioidal strain, spiral strain, affine shear, and reflected shear, were used to construct transformation sequences for the five patients in the actual growth sequences. Each transformation was applied to a coordinate space in which the facial profiles had been located. The profiles were oriented such that the origin of the coordinate system coincided with the top orb of the ear hole and the Frankfurt horizontal coincided with the x-axis. (See Pittenger et al., 1979, Footnote 2, for a discussion of the rationale governing this choice of origin and orientation.)

The youngest profile from each of the five actual growth sequences was taken as the profile to which all transformations were applied. The maximum value of the free parameter for each of the transformations was determined by finding the amount of each transformation that produced a change in facial angle corresponding to the magnitude of change effected by actual growth. Three intermediate profiles were produced by applying fractional amounts (1/4, 1/2, 3/4) of the maximum value of the free parameter to the original, untransformed profile. These fractional values assigned to the free parameter changed the facial angle by amounts similar to the changes measured on the intermediate profiles in the actual growth sequences. Thus, four transformation sequences, each consisting of five profiles—the original untransformed profile and four transforms—were constructed for each of the five patients for whom actual growth sequences had been obtained.

Stimulus preparation. All transformations were performed by computer. The graphic output of these transformations was produced on an electrostatic printer/plotter, and the resulting hard copy was then xeroxed in order to produce the stimuli shown to subjects. The actual growth sequence and the five transformation sequences for Patient BB are shown in Figure 2.

For the actual growth sequences, the five longitudinal profiles were arranged on a single page, with the youngest profile appearing at the far left and successively older profiles appearing to its right. A similar procedure was followed for the transformation sequences: The youngest profile in the actual growth sequences, from which all of the transform profiles had been derived, was placed on the left side of the page followed by the four

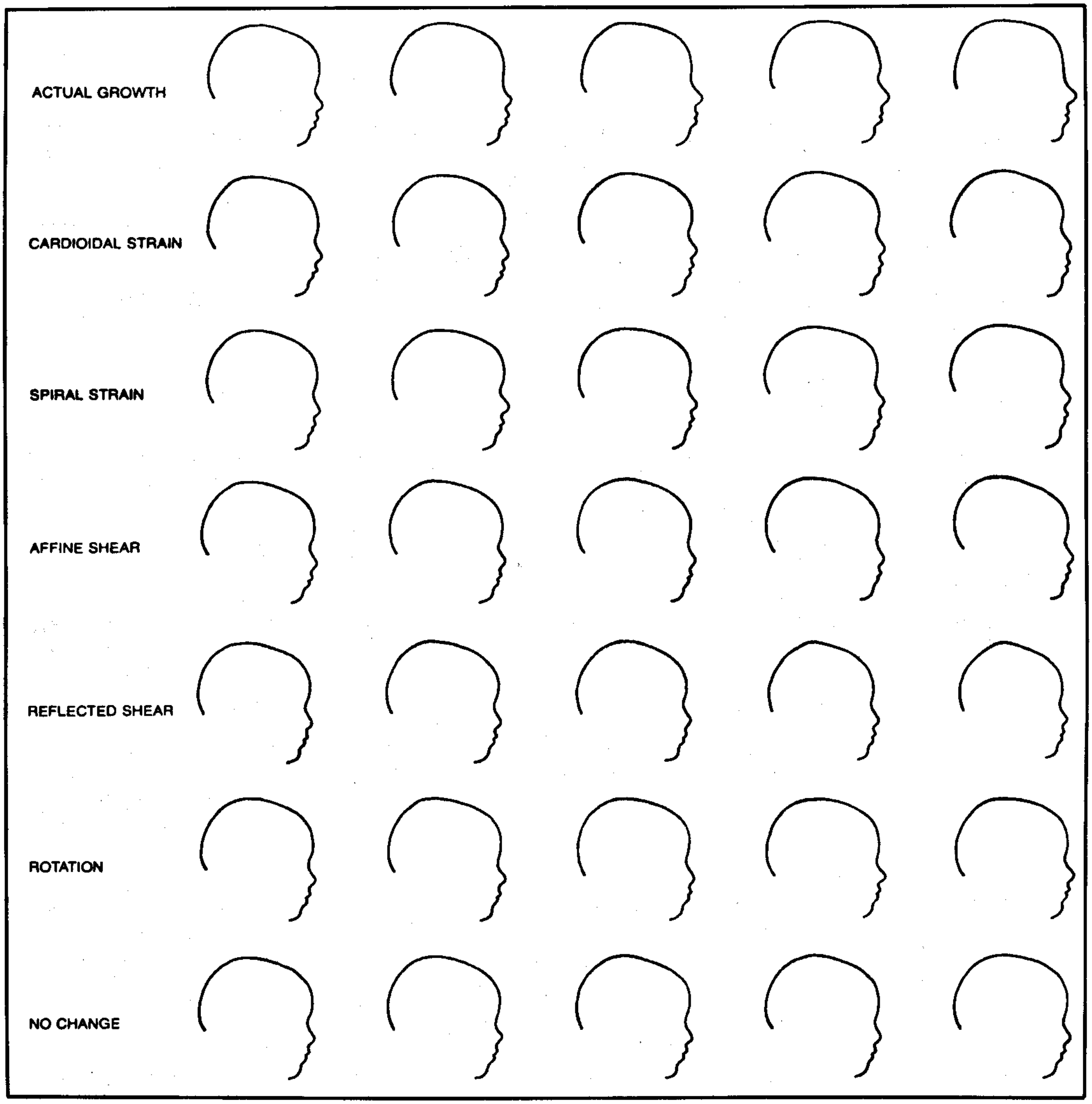

Figure 2. Examples of profile sequences resulting from the four prospective growth transformations, actual growth, rotation, and no change.

profiles resulting from successively greater amounts of the given transformation.

In addition to these sequences, three types of "nongrowth" control sequences were used to determine that subjects would not describe a transformation as "growth" regardless of whether it was a reasonable model of the actual event: (a) Five "no-change" sequences were constructed in which the five profiles were identical; (b) five "rotation" sequences were employed in which the initial profile was rotated by the same number of degrees as the change in facial angle; (c) ten sequences were formed by using large amounts of nongrowth transformations, such as rotations of 90° and 360°, and linear strain ($y' = ay$, $x' = bx$), which compresses the head. All profiles in each sequence were size normalized by equating their arc lengths.

Subjects. Forty-nine undergraduates at the University of Connecticut participated in the experiment for course credit. The data of nine students were eliminated because the students had prior knowledge about the experiment; this resulted in a final total of 40 subjects.

Procedure. Two subjects viewed each of 20 randomizations of the 46 test sequences. Subjects were tested double-blind: A student-experimenter, who had no prior knowledge of the project, was given only instructions on the experimental procedure and was told to read the directions to subjects. The subjects were also uninformed about the purpose of the experiment or its concern with growth and aging. Moreover, the data of any subject, who, after the experiment, admitted knowing anything about the project prior to the testing session, was excluded from the final analysis. For this reason nine subjects had to be eliminated from the experiment.

The student-experimenter read the following set of carefully piloted instructions to subjects:

> Each of you has been given a test booklet and a set of answer cards. Please do not look at the booklet until you are instructed to begin. On the top answer card, write your name, date, and the number found on the cover of your test booklet (the randomization number). Please set this card aside until the end of the experiment. There is a separate answer card for every page in your test booklet. Each page in the test booklet contains a number at the upper right-hand corner and a set of five profiles of human heads arranged across the page. For each page begin by copying the page number on the front of your answer card (E points to the front). Then, examine the set of faces arranged on that page and determine whether there is a change among the faces as you view them from left to right. In some sequences there is no change—the five faces will be identical. If there is no change among the items as you view them from left to right, then write *no change* on the back of your answer card.
>
> On the other hand, if you observe a change across the sequence, try to view the five profiles as if they are of the same object at different moments in time. Your task is to identify the process that produced the change in the heads. That is, what process changed the faces on the left into those on the right?
>
> This task requires that your answers be brief, often of only one or two words in length; rarely should your responses exceed three words. For example, the appropriate form of your answers should be something like *gaining weight, expressing an emotion,* or *changing facial expression.*
>
> Lets look at the first practice example. Could someone suggest a name for the process that produced the change? [A good answer would be something like "rotation" or "falling back."] How about the second example? [No change.]
>
> Let me remind you that your job is to name the process or type of change that produced the sequences of heads. That is, what kind of change would turn the head on the left into each of the successive transforms to its right? You should *not* be describing how the appearance of the face has changed. For example, you should not say things such as *heads become larger at the top, heads become flater, heads become more pointed, nose becomes wider and less tall.* Just describe the process or transformation that might have produced the type of change observed in the sequences of heads. If you have any questions during the test about whether a response is appropriate, please ask me. Also, if you have difficulty labeling the process, do your best to indicate what might be the cause of the change.

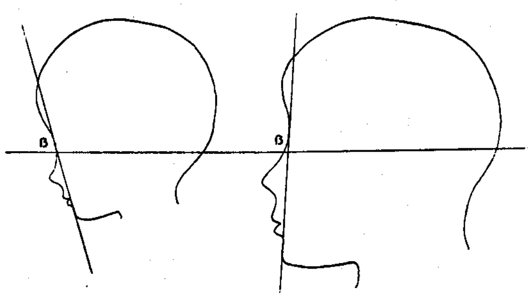

Figure 3. An illustration of how the facial angle (B) increases as a result of growth. (Our measure of facial angle is defined by the intersection of two lines, a line passing through the top orb of the ear hole and the bottom orb of the eye socket, called the Frankfort horizontal, and a line connecting the most prominent part of the chin with the deepest part of the depression just above the nose.)

Table 1
Age and Facial Angle of the Actual Growth Profiles for the Five Patients Whose Profiles Were Used

Patient	Profile 1	2	3	4	5
AA					
Age	5,5	9,4	13,3	16,0	25,5
Facial angle	89.0	91.0	92.5	94.0	95.0
BB					
Age	5,9	9,3	11,9	14,9	21,4
Facial angle	84.0	85.0	88.5	91.0	93.0
CC					
Age	6,9	10,9	13,9	18,0	21,0
Facial angle	84.0	85.0	87.0	88.5	90.0
DD					
Age	4,3	9,9	12,9	15,9	21,6
Facial angle	85.0	88.0	91.0	92.5	93.0
EE					
Age	6,3	9,3	12,9	15,9	21,0
Facial angle	88.0	89.5	91.0	93.0	95.0

Note. Age is expressed in years, months; facial angle, in degrees.

The pattern of results obtained on this unprompted free response experiment seems to depend on these lengthy instructions. Without such specific instructions, subjects tended to describe the effects of change rather than the process that produced the sequences of heads. And even with these elaborate instructions, subjects still reported the free response procedure to be fairly difficult.

Immediately after completing the free response task, subjects were asked if they had any prior knowledge about the nature or purpose of the experiment.

Scoring procedure. Two students were paid to categorize the linguistic labels given to the profile sequences. These judges, working independently, sorted the responses into two groups: If the response referred to "growth" or "aging," the judges placed it in the growth category; all responses that did not refer to growth were placed in the nongrowth category. Both judges rated the entire set of responses for the 40 subjects who participated in Experiment 1. The two judges disagreed on only 1 response out of 1,800. A third judge settled the disagreement.

Results and Discussion

The mean percentages of growth responses for the various types of profile sequences—actual growth, the four prospective growth transformations, rotation, and no change—are presented in Table 2. These data reveal that cardioidal strain and spiral strain, which satisfy the three invariants listed in the introduction, were seen as growth considerably more often than any of the other transformations. Affine shear and reflected shear, which meet fewer growth criteria, were perceived as growth far less often and only slightly more often than the nongrowth control sequences, rotation and no change, which elicited growth responses less than 1% of the time. These latter results provide especially strong evidence that subjects did not label a sequence as growth unless it resembled the actual style of change; apparently, the free response task did not force subjects to label a sequence as growth. This general pattern of growth responses was observed on each of the five patients whose profiles were used to construct the stimulus sequences. Thus, the general superiority of cardioidal strain over the other prospective growth transformations was not specific to a single patient or even a subset of patients.

However, these findings also show that none of the transformations resulted in nearly as many growth responses as the actual growth sequences, though cardioidal strain clearly provided the best fit to actual growth; cardioidal strain, $\chi^2(1) = 7.40$; spiral strain, $\chi^2(1) = 25.03$; affine shear, $\chi^2(1) = 101.40$; reflected shear, $\chi^2(1) = 108.52$; rotation, $\chi^2(1) = 176.65$; no change, $\chi^2(1) = 180.27$.[4] An examination of the percentages of growth responses for the five actual growth sequences (Table 2) reveals that much of their superiority over cardidal strain resulted from the relatively high percentage of growth responses to two of the five patients (AA, 85.0%; DD, 77.5%). An orthogonal comparison (N) of sequences AA/DD versus BB/CC/EE revealed that the presence of a large nasal protuberance had a significant effect on the number of growth labels, $F_N(1, 195) = 17.80, p < .001$. Careful scrutiny of both patients' actual growth sequences by three observers with some training in growth research showed two especially pronounced characteristics of adulthood: By the age at which his final profile was taken, Patient DD had developed a pronounced nasal sinus, which was larger than that for any other patient; and both Patient AA and Patient DD had relatively large noses, though two other patients had noses that were nearly as large.

While normal aging processes characteristically result in a relative enlarging of the nose and nasal sinus, the cardioidal strain transformation does not produce either change. Although these effects have been observed as general characteristics of craniofacial changes after puberty (Posen, 1967), it should be emphasized that a large variation exists across patients with respect to the size of the nose or the protuberance of the nasal sinus relative to the size of the facial mask. This suggests that a marked development of the nose and nasal sinus (as observed for Patients AA and DD) is not a universal characteristic of growth. Nor can it be argued that such pronounced characteristics are necessary for the perception of growth, as subjects still labeled the three remaining actual growth sequences and some of the cardioidal strain sequences as growth more than half of the time. Nonetheless, the actual growth sequences do contain information about the increase in age-level which is not found in the transformed profiles pro-

---

[4] The use of chi-square statistics throughout this study is intended only as a descriptive measure of the "goodness-of-fit" of subjects' perception of a prospective growth transformation to actual growth. As such, a test of statistical significance is inappropriate: We want to determine which transformation (of those tested) provides the best description of craniofacial growth. To use a significance test to determine whether one transformation is perceptually equivalent to actual growth would suggest that the significance test is a measure of perceived shape changes—a use for which statistical significance tests were never intended.

Table 2
Mean Percentage of "Growth" Labels on the Uninformed Free Response Task (Experiment 1) as a Function of Transformation (Including Actual Growth) and the Patient Whose Profile Was Used

	Patient					
Transformation	AA	BB	CC	DD	EE	*M*
Actual growth	85.0	55.0	55.0	77.5	47.5	64.0
Cardioidal strain	45.0	60.0	45.0	47.5	47.5	49.0
Spiral strain	32.5	42.5	32.5	30.0	35.0	34.5
Affine shear	10.0	30.0	12.5	10.0	7.5	14.0
Reflected shear	15.0	15.0	10.0	12.5	10.0	12.5
Rotation	2.5	0	0	0	2.5	1.0
No change	0	0	2.5	0	2.5	1.0
M	27.1	28.9	22.5	25.4	21.8	25.1

duced by cardioidal strain. This additional information may be responsible for the difference in growth responses between the cardioidal strain and actual growth sequences. But still, why should these aging characteristics, which start to develop only toward the end of the growth period and are confined to the oldest profile of the sequence, have such an important effect on the overall response level to the actual growth sequences?

An answer is suggested by the difficult (conservative) conditions under which this free response task was conducted: After completing the testing session, many subjects remarked that it had been extremely hard to pick up on any style of change when the facial profiles were arranged spatially across a page (cf. Peterson, 1974). Furthermore, subjects noted that age-level rarely stood out as an immediately obvious dimension of change on such profile outlines, perhaps because subjects were not "tuned up" to consider growth or aging as possible responses. In this situation the additional information provided by a bulging nasal sinus or a relatively large nose may have prompted subjects to notice the difference in age-level across the profile sequence. Task difficulty might also have been responsible for the surprisingly low absolute percentages of growth responses; even the actual growth sequences were described as aging less than two thirds of the time. If this conjecture is correct, then suggesting growth as an appropriate response in the instructions to the free response task should result in a marked increase in the percentage of growth responses for the actual growth sequences and for any transformation that provides a good description of growth. Experiment 2 tests this prediction.

Experiment 2: The Informed Free Response Task

Method

Stimuli. The same profile sequences used in Experiment 1 were employed in the second experiment.

Subjects. Forty undergraduates at the University of Connecticut participated in the experiment for course credit.

Procedure. In contrast to Experiment 1, the subjects were informed at the start of the testing session that the project was concerned with the perception of age-level and that the purpose of this specific experiment was to compare several proposed models of growth. Otherwise, the procedures and instructions were nearly identical to those used in Experiment 1, modified only by inserting "aging" and "growth" as examples of appropriate responses.

Scoring procedure. As in Experiment 1, two students were paid to sort the subjects' responses into growth and nongrowth categories. For the present experiment, there was perfect agreement between the two raters. All growth responses on the second experiment had occurred on the preceding experiment.

Results and Discussion

If the findings from Experiment 1 are indicative of the fact that subjects' response repertoires were relatively unconstrained, then on this prompted free response task, the absolute percentage of growth responses to any sequence that depicts growth should increase sharply, compared with that of the first experiment, and the percentages of growth responses to cardioidal strain, spiral strain, and actual growth should be more nearly equal. The results of Experiment 2 agree with both predictions (Table 3).

As in the preceding experiment, for each of the five patients the cardioidal strain and spiral strain sequences were described as growth far more often than any of the other transformations, and both of these transformations again provided better fits to actual growth than the other prospective growth transformations: cardioidal strain, $\chi^2(1) = .44$; spiral strain, $\chi^2(1) = 27.04$; affine shear, $\chi^2(1) = 196.28$; reflected shear, $\chi^2(1) = 256.10$; rotation, $\chi^2(1) = 292.77$; no change, $\chi^2(1) = 322.27$. But unlike Experiment 1, the percentages of growth responses to the cardioidal strain (89%, $SD = 5.53\%$) and actual growth (91%, $SD = 5.31\%$) sequences were almost identical and nearly reached the maximum level. Moreover, there was an interaction between the transformations and the experimental instructions (Experiment 1 vs. Experiment 2), $F(6, 468) = 24.22$, $p < .001$: Even though the percentage of growth responses to affine shear rose by 8% over that in Experiment 1, that increase was considerably smaller than the increments for cardioidal strain (40%), spiral strain (35%), or actual growth (27%); the changes in growth responses for the reflected shear, rotation, and no-change sequences were all less than 1%.

Thus, the outcome of Experiment 2 suggests that the low levels of growth labels to the actual growth and cardioidal strain sequences in the first experiment resulted, at least in part, from the fact that subjects did not have the appropriate repertoire of labels at their (ready) disposal. When "growth" or "aging" were suggested to subjects as appropriate responses, the percentages of growth labels to the actual growth, cardioidal strain, and spiral strain sequences increased sharply. Additional support for this conclusion is provided by an orthogonal comparison of the two actual growth sequences with pronounced noses, Patients AA and DD, with the remaining actual growth sequence, $F_N(1, 195) = 1.25$, $p > .05$. The effect of the pronounced noses on subjects' growth labels diminished sharply, compared with that in Experiment 1 (cf. Tables 2 and 3). This suggests that to a large extent the differences between cardioidal strain and actual growth in the first experiment are likely a consequence of "instructional tuning" (prompted vs. unprompted free response tasks).

Experiments 1 and 2 show that cardioidal strain and spiral strain capture the perceived information for craniofacial growth better than do the other prospective growth transformations, but there are changes that occur during actual growth which are not produced by either transformation. Under sufficiently difficult conditions, this additional information in the actual growth sequences can have significant perceptual consequences

Table 3
Mean percentage of "Growth" Labels on the Informed Free Response Task (Experiment 2) as a Function of Transformation (Including Actual Growth) and the Patient Whose Profile Was Used

	Patient					
Transformation	AA	BB	CC	DD	EE	*M*
Actual growth	92.5	87.5	90.0	95.0	90.0	91.0
Cardioidal strain	90.0	85.0	85.0	90.0	95.0	89.0
Spiral strain	87.5	70.0	62.5	65.0	65.0	70.0
Affine shear	22.5	20.0	27.5	12.5	27.5	22.0
Reflected shear	5.0	22.5	7.5	12.5	7.5	11.0
Rotation	10.0	5.0	2.5	7.5	2.5	5.5
No change	5.0	0	0	2.5	0	1.5
M	44.6	41.4	39.3	40.7	41.1	41.4

(Experiment 1). On the other hand, the results of Experiment 2 indicate that the changes modeled by cardioidal strain and spiral strain can specify craniofacial growth. Thus, the present study demonstrates that the cardioidal strain transformation, when applied to profiles of human heads, effects a style of change that is seen as craniofacial growth.

Experiment 3: The Growth Rating Tasks

Before elaborating on the outcomes of Experiments 1 and 2, the outcome of a task using a nonlinguistic dependent measure should be considered to demonstrate that this basic pattern of results is not an artifact of the linguistic ambiguity inherent in the free response procedure. For this purpose a rating task was administered to the subjects who participated in the first two experiments.

Method

Stimuli. The profile sequences from Experiments 1 and 2 were used in this experiment.

Subjects. The 80 subjects who participated in Experiments 1 and 2 were tested in the present experiment.

Procedure. After completing the free response task, the subjects in Experiment 1, who had been tested double-blind, were informed of the experiment's purpose by a second experimenter. They were also asked, at this point, if they had any knowledge about the experiment prior to the session. The data for nine subjects were eliminated from the analysis for this reason.

Subjects from both experiments were then instructed to rate each of the sequences used in the free response task on the basis of its resemblance to growth. A scale ranging from 0 to 4 was used for this task, with 0 indicating that the sequence did not look like actual growth and 4 that it did. The subjects were also told to identify the *younger* end (left or right) of each sequence that was not rated 0.

Scoring procedure. For all profile sequences, the younger end was intended to be on the left. This allowed us to incorporate the judgments of the younger end of the profile sequences into the numerical ratings of "how much like growth each sequence appeared" by negating the numerical estimate as a negative value if the younger end of the sequence was reported to be on the right.

Results and Discussion

The purpose of Experiment 3 was to replicate the basic pattern of results obtained on the free response tasks (Experiments 1 and 2) by using another experimental procedure. Subjects, who participated in the two preceding experiments, were given a rating task in which they indicated how much like growth each of the sequences appeared. As subjects had to be told about the relevance of growth in order to perform the task, one might assume that all participants were similarly "tuned up" to the rating task. It was surprising, however, that there were informative differences on the rating task between subjects from the first and those from the second experiment.

The mean ratings for each sequence type are given separately for subjects from Experiments 1 (Group E3-1) and 2 (Group E3-2; Tables 4 and 5). For both groups of subjects, the cardioidal strain and spiral strain sequences were rated as more like growth than any of the other transformations, and only rarely was the right end of the sequence incorrectly reported as younger for either transformation (cardioidal strain: Group E3-1, 2.0%; Group E3-2, 2.0%; spiral strain: Group E3-1, 5.5%; Group E3-2, 3.0%). The mean scores for the affine and reflected shears were only marginally above zero; this was due, only in part, to subjects' tendency to see the younger end of the sequence on the right (for affine shear, Group E3-1: 35.5%; Group E3-2: 14.0%; for reflected shear, Group E3-1: 32%; Group E3-2: 12.5%), a situation that negated the numerical rating. Finally, although the mean ratings for the rotation and no-change control sequences were marginally above zero, considerably fewer ratings had to be negated (for rotation, Group E3-1: 11.5%; Group E3-2: 1.5%; for no change, Group E3-1: 12.0%; Group E3-2: 2.0%) than for the two shear transformations.

As in the free response experiments, the ratings for cardioidal strain provided the best fit to actual growth for both groups of subjects (Group E3-1 [8 *df*]: cardioidal strain, $\chi^2 = 69.64$; spiral strain, $\chi^2 = 126.17$; affine shear, $\chi^2 = 247.93$; reflected shear, $\chi^2 = 199.27$; rotation, $\chi^2 = 289.67$; no change, $\chi^2 = 291.82$; Group E3-2 [8 *df*]: cardioidal strain, $\chi^2 = 5.86$; spiral strain, $\chi^2 = 34.62$; affine shear, $\chi^2 = 239.93$; reflected shear, $\chi^2 = 317.30$; rotation, $\chi^2 = 383.74$; no change, $\chi^2 = 381.64$).

Table 4
Mean Growth Ratings by Subjects Who Participated in Experiment 1 as a Function of Transformation and Patient Whose Profile Was Used

	Patient					
Transformation	AA	BB	CC	DD	EE	*M*
Actual growth						
M	3.78	3.28	3.45	3.78	2.88	3.43
SD	.72	1.55	.94	.65	1.85	1.14
Cardioidal strain						
M	2.13	2.88	2.40	3.00	3.08	2.70
SD	1.65	1.01	1.32	1.10	1.17	1.25
Spiral strain						
M	1.28	2.33	1.53	2.33	2.35	1.96
SD	1.73	1.13	1.43	1.88	1.33	1.50
Affine shear						
M	.55	−.90	−.68	.18	.28	.12
SD	1.87	1.95	2.10	2.27	1.90	2.02
Reflected shear						
M	.80	.23	−.55	−.78	1.60	.39
SD	1.59	2.39	1.90	2.77	2.13	2.16
Rotation						
M	0	.03	.08	.05	.15	.06
SD	.92	1.05	1.51	1.45	1.32	1.25
No change						
M	0	.03	.08	.05	.15	.06
SD	.22	.27	.41	.22	.36	.30
M	1.29	1.24	.90	1.34	1.58	1.27
SD	1.24	1.33	1.37	1.48	1.44	1.37

However, the mean difference in the ratings for the cardioidal strain and actual growth sequences was much greater for subjects who participated in the double-blind free response task (1.65) compared with the subjects from the second experiment (.38); the interaction between transformation and experimental groups (E3-1 vs. E3-2) was significant, $F(6, 468) = 10.95$, $p < .001$, for the same reason as in the free response experiments (Table 4; cf. Table 2): Subjects from Experiment 2 gave much higher ratings to the cardioidal and spiral strain transformations than did the subjects from the first experiment, whereas the ratings for the remaining transformations changed by smaller amounts. Thus, it appears that subjects' performance on this rating experiment was affected by the instructions used to tune them to the free response task performed earlier, that is, previous contact with the profile sequences had created a significant "perceptual bias" on the rating task. Perhaps those subjects from Experiment 1, who, it might be conjectured, were more likely to rely on the additional source of age-level information—the size of the nose or nasal sinus—continued to do so on the rating task; for this reason, the actual growth sequences may have continued to look more like aging than the cardioidal strain sequences. In contrast, the subjects from the second experiment, whose free responses were apparently less biased by this information, allowed it to have only a minor effect on their performance on the rating task; this explains the more similar ratings for cardioidal strain and actual growth. In support of this speculation, we might note that subjects from the first experiment assigned the highest mean ratings on the actual growth sequences to Patients AA and DD, $F_N(1, 195) = 9.94$, $p < .001$,

Table 5
Mean Growth Ratings by Subjects Who Participated in Experiment 2 as a Function of Transformation and Patient Whose Profile Was Used

	Patient					
Transformation	AA	BB	CC	DD	EE	*M*
Actual growth						
M	3.83	3.63	3.73	3.88	3.73	3.76
SD	.59	.76	.77	.33	.59	.61
Cardioidal strain						
M	3.45	3.70	3.63	3.63	3.75	3.63
SD	1.63	.75	1.04	.91	.62	.99
Spiral strain						
M	3.68	2.78	3.25	2.98	2.85	3.11
SD	.61	1.57	1.30	1.41	1.57	1.29
Affine shear						
M	1.25	.93	.73	.45	.75	.82
SD	1.59	1.33	1.67	1.64	1.76	1.60
Reflected shear						
M	.10	.38	.23	.28	.20	.24
SD	.74	1.34	1.11	1.43	1.15	.95
Rotation						
M	.05	0	0	.05	.05	.03
SD	.38	0	.22	.31	.22	.23
No change						
M	.03	0	.03	.05	.20	.06
SD	.15	.22	.35	.31	.46	.30
M	1.77	1.63	1.66	1.62	1.65	1.66
SD	.81	.85	.92	.91	.91	.85

the same patients for whom the largest percentages of growth responses were given in Experiment 1. On the other hand, the subjects from Experiment 2 assigned more nearly equal ratings to the five actual growth sequences, $F_N(1, 195) = 6.40$, $p < .05$ (but cf. Table 5), though the ratings for Patients AA and DD were slightly higher than for the other patients; perhaps they, too, were sensitive to this information.

General Discussion

The research described in the present article was based on the hypothesis that a style of change is uniquely specified by those geometric relations that are invariant over all structures seen as undergoing that style of change. Three experiments examined some implications of this hypothesis for the perception of craniofacial growth. The experiments were based on previous demonstrations that a group of transformations called cardioidal strain is perceived as growth in a variety of different contexts (Pittenger & Shaw, 1975; Pittenger et al., 1979). Cardioidal strain preserves the angular coordinate of every point on an object within a polar coordinate system, it preserves bilateral symmetry across the vertical axis, and it preserves the continuity and direction of curvature of all contours except along the vertical axis. Because of the finding that cardioidal strain is perceived as growth, we made three specific predictions: First, the biological processes of actual growth would preserve the same invariants; second, any other group of transformations that preserves those invariants would also be perceived as growth; and third, any group of transformations that does not preserve those invariants would be perceived as a style of change that is qualitatively different from growth.

The last of these predictions was clearly supported by the results of each experiment. All of the transformations that did not preserve the same invariants as cardioidal strain were seldom labeled as growth in free response tasks (Experiments 1 and 2) and were generally assigned the lowest possible rating on a growth rating task (Experiment 3). Cardioidal strain, spiral strain, and actual growth produced a much higher percentage of growth-related responses in every condition. This pattern of results was obtained for more than 85% of the individual subjects who participated in the different experiments.

The other predictions were only partially confirmed, however, since there were definite differences between subjects' responses to cardioidal strain, spiral strain, and actual growth. One finding in particular that had not been anticipated is that the actual growth of human craniofacial soft tissue is not always bilaterally symmetrical. Sometimes, asymmetries may be introduced as a result of a disproportionate increase in size of the nose relative to other parts of the face, or a bulging of the brow above the sinus cavities. These asymmetries are more clearly visible in some individuals than in others, and their perceptual salience is probably affected by attentional factors. In two of the actual growth sequences used in the present experiments, for example, there was a noticeably pronounced enlargement of the nose and nasal sinus. This did not affect the responses of one group of subjects (Experiment 2 and Group E3-2 in Experiment 3) whose initial instructions for the free response task specifically mentioned growth as a possible category of change, but it did affect the responses of a second group of subjects (Experiment 1 and Group E3-1 in Experiment 3) who were given free response instructions in which all references to growth had been deleted.

A second finding that had not been anticipated was the consistent difference between cardioidal strain and spiral strain. Although both of these transformations preserve the same invariants, cardioidal strain was responded to as growth significantly more often than was spiral strain in every condition. One possible explanation of this finding is that the three invariants, which are shared by the two transformations, define a class of "growth transformations" that approximates actual human growth. It is not necessary that all transformations within the class are perceived identically as long as they are more readily perceived as growth than is any transformation that is not a member of the growth class. Of course, this does not explain how a perceiver distinguishes among different transformations within the growth class, or why one transformation more closely resembles growth than another. Our current analysis based on three geometric invariants postulated to hold for growth cannot as yet provide the answers to these questions. Consequently, further analysis of growth events is called for to determine whether additional geometric invariants exist by which the cardioidal and spiral strain transformations, which are perceptually distinguished, might not also be theoretically distinguished.

It is interesting to note that there is independent evidence that the three invariants that appear to be specific to craniofacial growth are theoretically well motivated: From a few basic assumptions about the biomechanics of human growth, Todd et al. (1980) recently derived another transformation that preserves the same invariants as cardioidal strain and spiral strain, $\theta' = \theta$, $r' = r\ [1 + k\ (1 - \cos\ \theta)]$. This new transformation has been used to predict changes observed in longitudinal series of x-rays of human heads, and it is able to account for more than 80% of the variance for most individuals (Todd & Mark, 1981).

It should also be noted in passing that human growth is but a single example of a perceptually salient style of change. In order to demonstrate the theoretical utility of distinguishing styles of change by properties of objects that remain invariant, it is necessary to consider a wide range of examples. Thus, it behooves us to provide additional evidence that a group-theory approach to the perception of change can account for observers' abilities to recognize other styles of change, such as animate gaits, and to distinguish between rigid and nonrigid motion.

References

Cassirer, E. The concept of group and the theory of perception. *Philosophy and Phenomenological Research*, 1944, *5*, 1–35.

Cutting, J. E., Proffitt, D. R., & Kozlowski, L. T. A biomechanical invariant for gait perception. *Journal of Experimental Psychology: Human Perception and Performance*, 1978, *4*, 357–372.

Gibson, J. J. *The perception of the visual world*. Boston: Houghton Mifflin, 1950.

Gibson, J. J., & Gibson, E. J. Continuous perspective transformations and the perception of rigid motion. *Journal of Experimental Psychology*, 1957, *54*, 129–138.

Heider, F., & Simmel, M. An experimental study of apparent behavior. *American Journal of Psychology*, 1944, *57*, 243–259.

Johansson, G. Visual motion perception. *Scientific American*, 1975, *232*, 76–88.

Johansson, G. Perception. In M. R. Rosenzweig & L. W. Porter (Eds.), *Annual review of psychology* (Vol. 31). Palo Alto, Calif.: Annual Reviews Inc., 1980.

Mark, L. S. *A transformational approach toward understanding the perception of growing faces*. Unpublished doctoral dissertation, University of Connecticut, 1979.

Mark, L. S., Todd, J. T., Shaw, R. E., & Pittenger, J. B. Perceptual information for distinguishing styles of change. In R. Shaw & W. Mace (Eds.), *Event perception: An ecological perspective*. Hillsdale, N.J.: Erlbaum, in press.

McCammon, R. W. *Human growth and development*. Springfield, Ill.: Charles C Thomas, 1970.

Peterson, R. G. *The relationship between contour deformation and perceived separation of forms in depth*. Unpublished doctoral dissertation, University of Minnesota, 1974.

Pittenger, J. B., & Shaw, R. E. Aging faces as viscal-elastic events: Implications for a theory of nonrigid shape perception. *Journal of Experimental Psychology: Human Perception and Performance*, 1975, *1*, 374–382.

Pittenger, J. B., Shaw, R. E., & Mark, L. S. Perceptual information for the age-level of faces as a higher-order invariant of growth. *Journal of Experimental Psychology: Human Perception and Performance*, 1979, *5*, 478–493.

Posen, J. M. A longitudinal study of the growth of the nose. *American Journal of Orthodontics*, 1967, *53*, 746–756.

Shaw, R., McIntyre, M., & Mace, W. The role of symmetry theory in event perception. In R. MacLeod & H. Pick, Jr. (Eds.), *Studies in perception: Essays in honor of J. J. Gibson*. Ithaca, N.Y.: Cornell University Press, 1974.

Shaw, R., & Pittenger, J. Perceiving the face of change in changing faces: Implications for a theory of object perception. In R. Shaw & J. Bransford (Eds.), *Perceiving, acting, and knowing: Toward an ecological psychology*. Hillsdale, N.J.: Erlbaum, 1977.

Todd, J. T., & Mark, L. S. Issues related to the prediction of craniofacial growth. *American Journal of Orthodontics*, 1981, *79*, 63–80.

Todd, J. T., Mark, L., Shaw, R., & Pittenger, J. The perception of human growth. *Scientific American*, 1980, *242*, 106–114.

Received January 18, 1980 ■