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Distortions of three-dimensional space in the perceptual analysis of motion and stereo

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Abstract. A fundamental issue in the study of human vision is the accuracy with which observers can perceive the three-dimensional structures of objects in the environment. The formal mapping from physical to perceived space is considered as a geometric transformation, and the literature is reviewed to identify which aspects of object structure are systematically distorted by this transformation and which ones remain invariant. In the perceptual analysis of several different sources of optical information, including motion and stereo, both individually and in combination, there is a consistent pattern of results to indicate that perceived depth intervals are scaled differently from comparable intervals in either horizontal or vertical directions. These and other findings provide strong evidence that the relationship between physical and perceived space is noneuclidean, and that the three-dimensional structures of objects can appear systematically distorted even when viewed under full cue conditions.

1 Introduction

From the day-to-day experiences of human observers, it is obvious that patterns of visual stimulation can provide reliable information about the three-dimensional layout of the surrounding environment. From a theoretical point of view, however, this phenomenon is most perplexing, since the properties of optical stimulation appear to have so little in common with the properties of real objects in physical space. Whereas real objects are composed of tangible substances such as earth, metal, or flesh, the proximal stimulus for our perceptual awareness of objects consists of nothing more than flickering patterns of light. Nevertheless, for many animals, including humans, these patterns of light are the primary source of sensory information for perceiving the layout of surfaces in three-dimensional space.

One of the fundamental questions about human perception that has intrigued researchers for centuries is how our perceptual knowledge of the environment relates to its physical structure. There has been considerable debate surrounding this issue. Some investigators have argued that visual perception is essentially veridical, while others have insisted that it is inherently impoverished and subject to large distortions. Those who believe that perception is veridical tend to focus their arguments on its functional capabilities. If we could not accurately perceive the three-dimensional layout of the environment, then how could we successfully navigate around obstacles during locomotion, catch baseballs, drive automobiles, or perform any of the other complex visually guided behaviors of which we are capable? Those who believe that perception is impoverished, on the other hand, can point out the many perceptual illusions that have been documented over the years, including the perceived gyrations of a rotating trapezoidal window, or the figural illusions of lengths or angles such as the one shown in figure 1.

What complicates this debate is that the evidence to support both extreme positions is indisputable. There have been many well-controlled experiments to show that perceptual space can be systematically distorted, and many other equally good experiments to show that our perceptual judgments can be quite accurate. Norman and

Todd (1992) and Tittle et al (in press) have recently suggested a possible theoretical explanation for these seemingly conflicting results based on an analysis of geometric transformations first proposed by the German mathematician Felix Klein. Klein noted that any geometric transformation of an object will alter some of its structural properties, while leaving others invariant, and that it is possible to categorize different transformations into a hierarchy based on how they affect different aspects of object structure. Note that this same analysis can be applied to the geometric transformation between physical and perceived space. Those properties that are invariant over this mapping should be perceived with a high degree of accuracy, while judgments of properties that are not invariant should result in systematic errors. Let us now consider the patterns of performance predicted by this analysis for several possible relationships between physical and perceived space.

Suppose, for example, that physical and perceived space were congruent to one another. Our perceptions of three-dimensional structure in that case would be perfectly veridical. Since the geometry of physical space is euclidean at scales that matter to humans, we would expect that perceptual space should be euclidean as well. This would suggest in particular that perceived lengths should satisfy the Pythagorean theorem allowing them to be accurately compared at different positions and orientations.

Another possibility that has been suggested in the literature is that physical and perceived space are related by an affine transformation, such that intervals in depth appear systematically stretched relative to intervals in horizontal or vertical directions (eg see Wagner 1985; Todd and Norman 1991). An affine distortion of perceptual space would cause systematic errors in any perceptual judgment involving a comparison of lengths or angles of line intervals oriented in different directions, but it would have no effect at all on judgments of properties that are invariant under this type of transformation. Such properties include the coplanarity of a configuration of points, the parallelism of a pair of line segments, or the ratio of lengths for line segments oriented in the same direction.

There are many other more-complicated transformations by which physical and perceived space could potentially be related. One such category that we shall refer to as an ordinal mapping includes any transformation for which perceived depth increases monotonically with physical depth, such that relative depth order is preserved in any given visual direction (see Todd and Reichel 1989). If perceptual space were distorted in this way, observers would be inaccurate at judging the affine properties described above, but they might still be able to determine whether one point is farther away in depth than another, or to detect discontinuities of surface structure.

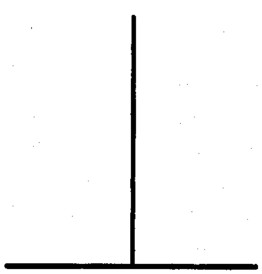

Figure 1. A simple configuration of two lines that demonstrates how perceptual space can be systematically distorted. Although the vertical line appears significantly longer, the two lines are actually of identical length. This illusion is due partially to the fact that verdical lines appear longer than horizontal lines, and partially to the configurational effects of the T-junction.

An important methodological implication of Klein's analysis of invariance under transformation is that all perceptual tasks are not equivalent for investigating the structure of perceptual space. For example, if one wishes to demonstrate that observers are sensitive to the euclidean metric structure of a planar surface, a distance-bisection task would be methodologically inappropriate, since that would only require a knowledge of three-dimensional structure up to an arbitrary affine transformation. To demonstrate the perception of euclidean structure, it would be necessary to show that observers can accurately compare length intervals in different positions and orientations.

There are several different criteria that can be used to evaluate the accuracy of observers' judgments for any given aspect of the three-dimensional structure of an object. One relevant factor to consider is their precision or reliability. This can be measured as a discrimination threshold or by the standard deviation for a given stimulus over multiple trials of adjustment or magnitude-estimation tasks. If these measures are normalized as dimensionless Weber fractions, it is possible to scale an observer's relative sensitivity to a wide variety of different three-dimensional properties (eg length, orientation, or curvature). A second relevant factor to consider is the existence of any systematic biases in observers' judgments. It is possible that the magnitude of a given stimulus property over multiple trials can appear consistently too large or too small, even if it is judged with a high degree of reliability. Still another important aspect of performance is the extent to which observers' perceptions exhibit constancy over changes in the position or orientation of an object. The issue of constancy is particularly relevant in evaluating the accuracy of observers' perceptions in natural vision, since objects in the real world are typically viewed from a wide variety of different vantage points.

It is important to keep in mind when examining the relationship between physical and perceived space that there are many potential sources of information available in natural vision, and the accuracy with which they specify different aspects of object structure need not be identical. At a theoretical level of analysis, for example, occlusions in an image can only specify ordinal depth relations, whereas other sources of information could in principle be used to specify the complete euclidean metric structure of an object.

Because it is generally believed that motion and stereo are the most powerful sources of optical information available in natural vision for the perceptual analysis of three-dimensional structure, these two modalities have been the most extensively investigated. During the past several years, in particular, there has been a growing body of research on the abilities of human observers to judge various aspects of three-dimensional structure from motion and stereo both individually and in combination. In the present article we will review some of the most recent findings on this topic. The paper is organized into four parts. First, we will consider the accuracy of perceived structure from motion for judgments of euclidean, affine, and ordinal properties. We will then describe how similar judgments are performed on the basis of convergence and binocular disparity. Next, we will examine any possible interactions that may occur when these different sources of information are available simultaneously, and, finally, we will consider the distortions of perceptual space that can occur under full cue conditions in a natural environment.

2 The perception of three-dimensional structure from motion

Although the importance of motion as a potential source of information for the visual perception of three-dimensional form has been recognized for more than a century (eg see Mach 1886/1962), it is only in the past two decades that theorists have begun to develop explicit computational analyses of how optical motion could be used to determine specific aspects of environmental structure. Many of these analyses have been designed to be used with arbitrary configurations of discrete points rotating in depth under parallel projection, such that corresponding images of each point can be identified over successive intervals of time. Given these conditions, it can be proven mathematically that a unique rigid euclidean interpretation can be obtained (up to a reflection in depth) provided that a display contains at least three distinct views of four noncoplanar points (Ullman 1977, 1979). If a motion sequence of an arbitrary configuration under parallel projection contains only two distinct views, than its three-dimensional structure can only be determined up to an affine stretching transformation along the line of sight (see Ullman 1977, 1983; Bennett et al 1989; Huang and Lee 1989; Todd and Bressan 1990; Koenderink and van Doorn 1991). If, however, a two-view sequence is viewed under sufficiently strong polar perspective, then the perspective distortions in its optical projection can be used to obtain a unique euclidean interpretation (Longuet-Higgins 1981).

Because the number of distinct frames in an apparent-motion sequence is such a critical factor in the computational analysis of three-dimensional structure from motion, there have been numerous psychophysical investigations that have involved examination of how the judgments of human observers are influenced by this factor. There is now considerable evidence that observers can obtain compelling kinetic depth effects from two-frame motion sequences presented in alternation with appropriate spatial and temporal displacements (see Lappin et al 1980; Doner et al 1984; Braunstein et al 1987; Todd et al 1988; Braunstein et al 1990), and more recent research has shown in addition that there is little or no improvement on objective-response tasks when the number of frames in a motion sequence is increased beyond two (Todd and Bressan 1990; Todd and Norman 1991; Norman and Lappin 1992; Liter et al 1993). Since two-frame sequences under parallel projection are theoretically inadequate to compute euclidean metric structure, it follows from these results that the perceptual analysis of moving displays may not involve an accurate computation of euclidean distance relations in three-dimensional space.

This conclusion has been confirmed, moreover, by other experimenters who have examined how precisely observers can discriminate various aspects of the three-dimensional structure of an object from its pattern of projected motion. On tasks that require a knowledge of euclidean structure, such as discriminating three-dimensional lengths or angles in random orientations, observers' thresholds can be as high as 25% or 30% (Todd and Bressan 1990; Todd and Norman 1991; Norman and Todd 1993)—almost an order of magnitude higher than those obtained for many other types of sensory discrimination. This is also the case for displays presented under strong polar perspective (Todd et al 1993), which could in principle provide additional information to that which is available under parallel projection. These results contrast sharply, however, with those obtained for discriminations of object properties that are invariant over affine transformations in depth, such as distance-bisection tasks (Lappin and Fuqua 1983) or discriminating planar from nonplanar configurations (Todd and Bressan 1990).

Similar evidence for an affine distortion of perceptual space in the analysis of optical motion can be obtained from observers' magnitude estimations or adjustments of various aspects of the three-dimensional structure of an object. For example, Tittle et al (in press) have recently shown that observers are quite accurate at adjusting the angle between two rotating planes so that they appear to be parallel to one another—a property that is invariant under arbitrary affine transformations. If, on the other hand, the task is changed so that the planes must be adjusted to appear orthogonal (ie a non-affine property), then observers make large systematic errors and their reliability is reduced by an order of magnitude from what is typically achieved for planar adjustments.

Similar systematic distortions of perceived shape from motion have been reported in numerous investigations for a wide variety of depicted objects, including cylinders (Todd 1984; Tittle et al in press), ellipsoids (Loomis and Eby 1988, 1989), dihedral angles (Braunstein et al 1993), sinusoidally corrugated surfaces (Todd and Norman 1991), or random configurations of points (Liter et al 1993).

One important consequence of having an affine distortion in the relationship between physical and perceived space is that euclidean distance relations should appear systematically altered when an object is presented at different orientations in depth. Suppose, for example, that perceived depths are overestimated by 30%, as has been reported by Todd and Norman (1991), and an observer is presented with a rotating ellipsoid whose instantaneous depth is 30% smaller than its width. The ellipsoid should appear at that moment as a sphere, but if the same object is viewed again after a 90° rotation about a vertical axis, its depth should appear 69% longer than its width. This predicted failure of orientation constancy from an affine distortion of perceptual space has in fact been reported by Tittle et al (in press) and by Braunstein and Liter (1993).

3 The perception of three-dimensional structure from binocular stereopsis

Although both Euclid and Leonardo observed that the right and left eyes project different images of the same three-dimensional configuration, it was Wheatstone (1838) who provided the earliest empirical evidence that depth is perceived when disparate planar line drawings are presented separately to each eye. Since those early investigations, binocular disparity has been considered one of the most salient sources of optical information about three-dimensional structure. It is important to keep in mind, however, that there is a nonlinear relationship between horizontal disparity and physical depth that varies with the interocular separation and the convergence angle. Thus, disparity per se cannot adequately specify the euclidean metric structure of the environment without being 'scaled' by some other source of information to identify its current functional relationship with physical depth.

This process of disparity scaling could in principle be accomplished on the basis of knowledge of the fixation distance (Foley 1980) or an analysis of vertical disparities (Mayhew and Longuet-Higgins 1982), but there is a considerable amount of evidence to suggest that the abilities of human observers to perform such a process in practice may be quite limited. Although research has shown that it is possible to determine the ordinal depths of stereoscopic targets with amazing precision—ie thresholds can be as low as 2 s arc (Ogle 1952)—performance is surprisingly poor when observers are asked to discriminate the relative magnitudes of depth intervals. Discrimination thresholds for judging stereoscopic depth intervals at a fixed viewing distance can be as high as 10%-20%, even for highly practiced observers (McKee et al 1990), and these thresholds can be increased still further if the line intervals to be judged on different trials are presented at variable viewing distances and orientations (Todd et al 1993).

In other research involving magnitude estimation or adjustment tasks it has been shown that perceived metrical structure from stereo is systematically distorted in addition to being imprecise. For example, in a recent experiment by Johnston (1991) observers were asked to judge the apparent eccentricities of cylindrical surfaces presented at different viewing distances. The results revealed that the extensions of these surfaces in depth were perceived correctly at distances of approximately 1 m, but that they appeared systematically compressed at longer distances and systematically expanded at shorter distances. Similar findings were also obtained by Tittle et al (in press) in a series of adjustment tasks involving cylinders, dihedral angles, and planar surfaces. It is especially interesting to note in this context that there is an extensive literature on distortions of stereoscopic space that dates back to the time of Hering and Helmholtz. Some of the classic results of this research, which have been replicated many times over the past century [see Foley (1980) for a more complete review], include the apparent curvature of the frontoparallel plane, the apparent deviation of parallel alleys in depth, and the systematic distortions of distance-bisection judgments —all of which vary with viewing distance.

It is important to recognize that many of these classical experimental paradigms involve judgments of affine properties (ie planarity, parallelism, and distance bisection). Thus, the finding that these properties are systematically distorted in the mapping between physical and perceived space for binocular vision indicates that this mapping cannot be affine, as appears to be the case for the perceptual analysis of structure from motion. Some researchers have argued that the perceived metric structure of the environment from binocular stereopsis is best described as a homogeneous Riemannian space of constant negative curvature (Luneburg 1947, 1948, 1950; Blank 1953, 1958, 1961), though an important experiment by Foley (1972) suggests that the curvature of this space may not be isotropic. An alternative view suggested by Ogle (1962) is that stereoscopic space is essentially nonmetrical, and that all we really get from binocular disparity is ordinal information about relative depths with only a vague sense of the magnitudes of depth intervals. This latter hypothesis would seem to be supported by the notoriously poor reliability and high discrimination thresholds typically obtained for judgments of stereoscopic distance intervals (eg see McKee et al 1990; Todd et al 1993; Tittle et al in press).

4 The interactions of motion and stereo

All of the evidence described thus far has indicated that the perceived structures of objects in space are systematically distorted when the available optical information is restricted to motion or stereo presented in isolation. For the case of structure from motion, physical and perceived space appear to be related by an affine transformation, but when objects are viewed stereoscopically, their affine properties appear systematically distorted as well.

It is important to keep in mind, however, that observers in natural vision almost always have access to multiple redundant sources of optical information, and it is theoretically possible that these multiple sources could interact with one another in such a way to provide a more accurate determination of three-dimensional structure than would be possible from any one presented in isolation. For example, when motion and stereo are presented in combination, Richards (1985) has shown that it is possible to compute the euclidean metric structure of an object from two stereo views of three or more points, independently of fixation distance or the interocular separation. Thus, whereas a two-view stereogram or apparent-motion sequence would be individually inadequate to specify euclidean metric structure, they could in principle be used in combination so that motion could scale disparity (or vice versa).

Early studies of perceived three-dimensional structure from stereo and motion appeared to suggest that similar interactions may occur in human vision. Wallach et al (1963) and Epstein (1968) examined this issue by using a telestereoscope to provide conflicting information from motion and stereo. They found that depth judgments from binocular stereopsis were recalibrated after adaptation with inconsistent motion information. However, Fisher and Ebenholtz (1986) later showed that these depth aftereffects were caused by changes in the relationship between convergence and accommodation rather than the result of motion recalibrating binocular stereopsis.

Another potential source of evidence that motion can scale stereo has been reported by Johnston et al (1991), using a task in which observers judged the apparent eccentricity (ie shape) of cylindrical surfaces. The results indicated that the perceived extension of these surfaces in depth was attenuated with increasing viewing distance for static binocular displays, but that there were no effects of viewing distance when motion and stereo were presented in combination. There were, however, some potential problems with their experimental design, in that the stereo and combined displays were presented at different slants, and that there were artifactual constraints imposed by always truncating the cylinders at perfect right angles to their central axes. When these confounding factors were eliminated in a more recent experiment by Tittle et al (in press), no interactions between motion and stereo were obtained. This result was replicated, moreover, for several different tasks including judgments of cylindrical shape, dihedral angles, and planar surfaces. In all of these cases, the overall pattern of results for the combined stereo and motion displays was indistinguishable from those obtained for stereo alone. A related finding that stereo dominates motion when the two sources of information are placed in conflict has also been reported by Rogers and Collett (1989) and Tittle and Braunstein (1993).

An even stronger test of observer sensitivity to euclidean metric structure from combined stereo and motion displays has recently been reported by Todd et al (1993). They asked observers to discriminate the relative three-dimensional lengths of stereoscopically presented line segments within moving configurations that were rotating in depth about a vertical axis. When these line segments were presented at a fixed viewing distance but with random orientations, the observers' length-discrimination thresholds were approximately 15%. When viewing distance was allowed to vary, however, these thresholds increased to over 25%—an order of magnitude greater than what is typically reported for length discriminations of parallel lines in the frontoparallel plane. There was also a strong bias in the observers' responses to judge closer segments in depth as longer, which is consistent with the similar effects of viewing distance that have been reported for magnitude-estimation tasks.

5 Perceived three-dimensional structure under full cue conditions

Although motion and stereo are generally considered to be the primary sources of optical information for the visual perception of three-dimensional form, there are many other aspects of optical stimulation that are known to be perceptually informative as well (eg the classical pictorial depth cues). It is useful to consider, therefore, how the accuracy of observers' judgments is affected when all possible sources of information are available simultaneously. In a recent attempt to address this issue, Norman et al (1993) measured observers' discrimination thresholds for the angular separation between the surface normals of designated point pairs on computer simulations of smoothly curved surfaces depicted with shading, texture, motion, convergence, and binocular disparity. Despite the presence of these many redundant sources of information, however, the observers did not reach threshold until the differences in angular separation to be detected were as large as 25%.

One potentially relevant source of information about three-dimensional structure that has been ignored in much of the literature on this topic is accommodative blur. When a real object is observed in natural vision at a reasonably close viewing distance, its surface regions at different depths cannot all be kept in focus at the same time. Frisby and Buckley (1992) and Buckley and Frisby (1993) have recently argued that these gradients of accommodative blur are important sources of information in natural vision, and that because this information is unavailable when pictorial displays are viewed, it is possible that experiments which employ such displays could provide a misleading estimate of observers' perceptual performance. To provide empirical support for this hypothesis, they have also shown that direct viewing of real objects can produce significantly different perceptual judgments from those obtained when the same objects are simulated in stereograms.

Although accommodative blur may be a perceptually salient source of information, it remains to be demonstrated that it is sufficiently powerful to provide an accurate specification of euclidean metric structure. For direct viewing of real objects, Frisby and Buckley (1992) and Buckley and Frisby (1993) report accurate numerical judgments of the shapes of cylindrical surfaces or the orientations of planar ground surfaces, but this level of performance could only be achieved after explicit training with feedback on the experimental stimuli. Such a procedure reduces the task of magnitude estimation to one of object discrimination, and would obscure any systematic distortions of perceived intervals in depth as are typically obtained by using stereoscopic displays.

Other research has shown that if observers are not given response training with feedback, there can be large distortions in the apparent structure of physical space even when a scene is viewed directly in natural vision. Of particular relevance with respect to the perception of euclidean metric structure are the recent experiments of Wagner (1985), Toye (1986), and Loomis et al (1992), in which observers judged the apparent distance intervals between targets on the ground in an open field with full illumination. The results obtained in all of these experiments revealed that perceived intervals in depth are systematically compressed relative to those in a horizontal or vertical direction.

One possible limitation of these experiments that deserves to be highlighted is that the targets to be judged were all placed at relatively large viewing distances of greater than 3 m, at which the effects of accommodation are likely to be minimal. In an effort to overcome this limitation, Norman et al (in press) have recently used a length-matching task to measure any perceptual distortions that may occur in near visual space. The standard stimuli were line segments of variable length and orientation presented on a display screen at a variable distance of 1 or 2 m from the point of observation. On each trial observers were required to match the length of the standard line by positioning a stylus in a specified direction from a reference point marked on a textured tabletop surface at which they were seated. The results revealed that observers' judgments of three-dimensional length are surprisingly inaccurate and unreliable. The RMS error for different observers ranged from 14% to 44% of the standard length. There were also systematic biases of adjusted length that varied with viewing distance, the two-dimensional orientation of the standard, and whether the adjusted length was made horizontally, vertically, or in depth.

6 Summary and conclusions

When considered as a whole, the research reviewed in the present article provides an impressive body of evidence that there is a noneuclidean relationship between physical and perceived space, such that the true three-dimensional structures of objects appear systematically distorted. These distortions can occur in the perception of structure from motion, the perception of structure from binocular disparity and convergence, the perception of structure from motion and stereo combined, and even when real objects are viewed directly at near or far distances in a natural environment under full cue conditions. In all of these cases, there is considerable evidence that perceived intervals in depth are scaled differently from comparable intervals in horizontal or vertical directions. For computer simulations of objects rotating in depth under parallel projection, or for real objects viewed from a relatively long distance, the evidence suggests that these scales remain in fixed proportions to one another so that affine properties remain invariant in the mapping from physical to perceived space. For objects viewed in near visual space, however, this relationship is more complex in that perceived intervals in depth appear more and more compressed with increasing viewing distance (eg see Baird and Biersdorf 1967).

Our discussion thus far has been focused exclusively on the formal mapping between physical and perceived space. It is also possible, however, to consider the intrinsic structure of perceptual space without making any comparisons with the corresponding structure of the external environment. One possible method of measuring this intrinsic structure would be to have observers estimate the three angles in a triangle. If perceptual space is euclidean, then the sum of these three judgments should on average equal 180°. If the sum is greater (or less) than 180°, it would indicate that perceived space is elliptic (or hyperbolic). For stereoscopic targets viewed in the dark, there is considerable evidence to indicate that the intrinsic structure of perceived space is hyperbolic (eg see Blank 1961; Foley 1972).

It is especially interesting to note in this regard that it would be mathematically possible for perceptual space to have a euclidean intrinsic structure even if it were related to physical space by a noneuclidean mapping. This hypothesis has in fact been proposed by Wagner (1985) for the perception of three-dimensional form under full cue conditions. According to this view, lengths and angles of surfaces in the environment are represented in an intrinsically euclidean perceptual space, but need not be represented accurately. The fact that we can judge length at all suggests there is some truth to this hypothesis. However, there are other important consequences of this type of representation that are clearly in conflict with our everyday experience. If objects were perceptually represented by geometric properties that are anisotropically distorted with respect to their true values in physical space, then the structure of the environment should appear to distort as we move about within it. The fact that this does not occur provides strong evidence that our immediate awareness of a rigidly stable environment cannot be based on an explicit representation of its euclidean properties.

Although there is a large body of data to indicate that perceptual space is systematically distorted, many researchers still find it difficult to accept this conclusion, because it seems to be incompatible with the high degree of precision that human observers can obviously achieve in performing complex visually guided actions. What is much less obvious, however, is what performance of these actions reveals about the metrical structure of perceived space. Consider, for example, the common tasks of avoiding obstacles during high-speed locomotion, braking a moving vehicle with the correct force to prevent an impending collision, or running to intercept a moving projectile. How could observers perform such activities without having accurate knowledge of the euclidean metric structure of the environment? It is interesting to note in this regard that all of these tasks have been theoretically analyzed to show how they could be performed on the basis of optical information without having any knowledge of distance relations whatsoever (eg see Lee 1976; Todd 1981; Regan et al 1995). Given the existence of these analyses, and the empirical support for their psychological validity, there is certainly no reason to assume without evidence that accurate knowledge of euclidean metric structure is in any way necessary for the accurate performance of visually guided actions.

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